Nominal term spread, real rate and consumption growth*

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Robust empirical evidence suggests that a steep slope of the nominal yield curve predicts an increase in future real activity. We show that the negative of the slope closely traces the variation in the ex-ante real rate. We then argue that the predictive content of the slope for real activity empirically encapsulates the intertemporal tradeoff that arises in a broad class of equilibrium models. As a key implication, the estimates of the elasticity of intertemporal substitution (EIS) from the aggregate Euler equation are severely downwardly biased, implying a negative aggregate EIS. A model with heterogeneous agents and limited market participation can reconcile the positive EIS found in micro data with the negative relationship between the ex-ante real rate and aggregate consumption and output.

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I. Introduction

The slope of the nominal term structure, defined as the spread between the long-term Treasury yield and the T-bill rate, features prominently in the macro-finance literature. Robust empirical evidence suggests that a steep slope is associated with a future increase in the real economic activity, consumption and output (e.g. Estrella and Hardouvelis, 1991; Harvey, 1988). However, the question as to the economic interpretation of this empirical regularity remains open. This paper provides an answer to this question by arguing that the negative of the slope is a valid measure of the variation in the ex-ante real rate. Thus, the predictive content of the slope for the real activity empirically captures the aggregate Euler equation—the relationship between the ex-ante real rate and consumption or output growth—that arises in a broad class of macro and asset pricing models. Yet, the sign of the empirical relationship runs counter to the standard economic logic whereby a high ex-ante real rate should be associated with high consumption (output) growth in the future. We explore the economic underpinnings and implications of this finding.

As an object of central importance in economics and finance, the ex-ante real rate (nominal short rate minus expected inflation) is not directly observed. We begin with a simple insight: While the variation in the slope of the nominal yield curve can arise from movements in the term premium, expected inflation and/or the ex-ante real rate, we empirically show that the latter clearly dominates the other two, and is the main driver of the predictive power of the slope for the real activity. Thus, the negative of the slope is a valid instrument for the ex-ante real rate. The key advantage of recognizing this fact is that our measure of the ex-ante real rate does not require any proxy for expected inflation and relies solely on asset prices. Therefore, it avoids problems associated with limited samples over which direct (survey-based) measures of inflation expectations are available, and also circumvents making auxiliary assumptions about the time series dynamics of inflation which have substantially changed over time.

We exploit this observation to obtain new and precise estimates of the aggregate elasticity of intertemporal substitution (EIS). Since the nominal term spread is available over long time spans and has favorable stationarity properties across different monetary policy regimes, we are able to estimate the aggregate Euler equation exploiting 134 years of data, covering different inflation regimes and monetary policy arrangements. We document that estimates of the EIS from the aggregate Euler equation are severely downwardly biased—our estimates are consistently negative around -0.5. Similarly, replacing aggregate consumption by aggregate output in the Euler equation yields negative estimates with a comparable magnitude. Furthermore, negative estimates of the EIS are not specific to the US economy but seem to be a prevalent feature present of the international data as well.

The negative relationship between the real interest rate and aggregate consumption or output growth has a number of implications for macro and asset pricing models. First, biased estimates of the aggregate EIS help explain why the behavior of the yield curve is hard to reconcile with standard representative-agent asset pricing models (Duffee, 2012). In particular, our results cast light on the surprising finding in the literature that consumption-based models calibrated with the aggregate EIS close to unity imply a short-term real interest rate that is negatively correlated with the real interest rate from the data, as shown in Canzoneri, Cumby, and Diba (2007).[1]

Second, the inverted relationship in the output Euler equation has major implications for models of monetary policy because the central bank reacts to aggregate output but only a subset of agents reacts to changing interest rates by trading in asset markets. The aggregate output Euler equation (IS curve) is the core of the dynamic general equilibrium models used in monetary policy analysis, e.g. the New-Keynesian models. The effectiveness of monetary policy is determined by the sensitivity of the aggregate output growth to changes in the policy rate. Therefore, the negative relationship between the ex-ante real rate and the aggregate

[1] Canzoneri, Cumby, and Diba (2007) compare the model-implied real rate to the ex-ante and ex-post real rate obtaining similar results for both.

output growth changes the effects of monetary policy and its optimality conditions discussed in Bilbiie (2008).

Despite its importance, the literature is not conclusive on the value of the EIS. On the one hand, estimates from aggregate data suggest that the EIS is close to zero (Hall, 1988; Campbell, 2003; Fuhrer and Rudebusch, 2004). On the other hand, evidence from micro data indicates that the EIS is likely to be above unity for households with substantial asset holdings (Vissing-Jorgensen and Attanasio, 2003; Gruber, 2006). We show that the negative estimates of the EIS in the aggregate data can be reconciled with the positive EIS estimates from household-level data in a model with heterogenous agents and limited market participation. The key mechanism that changes the sign of the relationship between the real rate and aggregate consumption growth operates through the interaction of two types of households. A fraction of households, rule-of-thumb consumers, do not participate in asset markets and consume their wage income every period (Campbell and Mankiw, 1989; Mankiw, 2000). The remaining households, savers, own all the productive assets in the economy. Savers have two sources of income: wages and dividends. Changes in the real interest rate directly influence only the consumption and labor supply of savers, but the intertemporal choices of savers then have an impact on the equilibrium real wage of rule-of-thumb consumers and thereby alter their consumption. Changes in the real wage also lead to a variation in profit margins of firms, affecting the dividend income and the consumption of savers. The resulting effect of the real interest rate shock on aggregate consumption is negative if the fraction of rule-of-thumb households is sufficiently high and the labor supply is sufficiently inelastic. Standard values for the structural parameters in the calibrated model imply a magnitude of the negative relationship between the aggregate consumption growth and the ex-ante real rate that is close to the estimates we find in the data.

Our work is related to several areas in the finance and macro literature. First, numerous papers analyze the empirical properties of the real interest rate (Mishkin, 1981; Fama and Gibbons, 1982; Evans and Lewis, 1995; Ang, Bekaert, and Wei, 2007). The overall message of these papers is that measuring the ex-ante real rate is complicated by the fact that both the persistence and the volatility of inflation in the US vary over time (Barsky, 1987; Cogley, Primiceri, and Sargent, 2010). Despite this evidence, it is common to construct the ex-ante real rate using linear projections of the ex-post real rate on a set of instruments, which under regime changes will lead to biased estimates. This problem becomes especially important in long samples that we study in this paper. Our findings, in contrast, do not rely on assumptions about the statistical process for inflation.

Second, while a large literature documents that the nominal term spread predicts real output growth (Estrella and Hardouvelis, 1991; Plosser and Rouwenhorst, 1994; Ang, Piazzesi, and Wei, 2006; Bordo and Haubrich, 2008) and consumption growth (Harvey, 1988), there is no consensus as to the economic mechanism behind this predictability. Recently, Kurmann and Otrok (2013) relate the variation in the term spread to news shocks about the total factor productivity. We instead make the link between the predictive content of the nominal term spread and the aggregate Euler equation and show the importance of this result for the estimates of the aggregate EIS.

Third, estimates of the EIS from aggregate data (e.g. Hansen and Singleton, 1982; Hall, 1988; Fuhrer and Rudebusch, 2004; Bilbiie and Straub, 2012) have sizeable standard errors and are usually close to zero. A variety of explanations for the low magnitude of the EIS estimates have been proposed: non-separability of non-durable and durable consumption (Ogaki and Reinhart, 1998), limited stock market participation (Guvenen, 2006), bounded rationality (Gabaix, 2012), or the structural break in the EIS (Bilbiie and Straub, 2012). Importantly, these studies provide arguments for why the EIS estimates are biased toward zero but models outlined in these papers cannot explain why parameter estimates are consistently negative. The lack of accurate parameter estimates from aggregate data has motivated the analysis of household-level data. Vissing-Jorgensen (2002) and Attanasio, Banks, and Tanner (2002) show that the EIS is close to unity for stockholders. Similarly, Gruber (2006) estimates the EIS to be larger than one in a sample of households paying substantial capital income tax. The negative sign of the EIS that we estimate suggests that the discrepancy between the micro and macro estimates of the EIS may be even larger than previously documented. We argue that the difference arises as a result of aggregation of heterogenous households. The model we present is consistent with both the EIS close to unity for households participating in asset markets and the negative relationship between the real rate and the aggregate consumption.

Finally, asset pricing literature usually interprets the variation in the slope as reflecting the time-varying bond risk premium, motivated by the evidence in Fama and Bliss (1987) and Campbell and Shiller (1991). Our decomposition of the slope reveals that the contribution of the ex-ante real rate component dominates that of the risk premium. This fact in consistent with the finding of Cochrane and Piazzesi (2005) that bond risk premia cannot be explained by the standard factors in the yield curve, and with the yield curve decomposition in Cieslak and Povala (2014a). We also document that the predictability of nominal bond returns by the term spread is inflated by the inclusion of the Volcker experiment (1979–1982). As such, the presence of time-varying risk premia does not invalidate the negative of the slope as an instrument for the ex-ante real rate.

The remainder of the paper is structured as follows. Section II reviews the predictability of real activity by the slope, decomposing the slope into expected inflation, ex-ante real rate and the term premium. Section III provides arguments for why the negative of the slope is an appropriate measure of the ex-ante real rate. Section IV estimates the EIS in aggregate data using the negative of the term spread as a measure of ex-ante real rate and discusses econometric and robustness issues. Section V shows that the negative EIS estimates can be rationalized in a general equilibrium model with limited asset market participation for standard values of structural parameters. Section VI discusses the key mechanism for obtaining the negative aggregate EIS and studies its implications for term structure modeling.

II. Motivating evidence: predicting output growth with the nominal term spread

The predictive content of the slope for consumption or output growth is well established in the literature (e.g. Estrella and Hardouvelis, 1991; Harvey, 1988).[2] The regression studied in the literature is specified as:

$$\Delta g_{t+k/4} = \alpha_0 + \alpha_1 \text{slope}_t + \varepsilon_{t+k/4}, \quad k \in [1, \dots, 8] \tag{1}$$

where $g_{t+k/4} \equiv \frac{4}{k}\ln\left(GDP_{t+k/4}/GDP_t\right)$ is the annualized k-quarter GDP growth and time is measured in years. We measure the slope as a difference between the ten-year nominal Treasury yield and the three-month T-bill yield. Panel A of Table I reports the estimation results for the period Q1:1972–Q4:2013 which confirm that the slope contains significant predictive content for output growth across all horizons. However, the economic mechanism behind this empirical link remains largely elusive. To better understand the nature of this relationship, we first decompose the slope into more basic components and then relate these to the output growth. We start with the one-period nominal interest rate:

$$i_t = r_t + E_t \pi_{t+1}, \tag{2}$$

where r_t is the one-period real rate and $E_t \pi_{t+1}$ denotes the corresponding inflation expectations. The n-period nominal Treasury yield can be decomposed as follows:

$$y_t^{(n)} = \frac{1}{n}\sum_{i=0}^{n-1} E_t r_{t+i} + \frac{1}{n}\sum_{i=1}^{n} E_t \pi_{t+1} + rp_t^{(n)}, \tag{3}$$

where $rp_t^{(n)}$ represents the risk premium. Consequently, the variation in the slope can be traced back to three economic sources: expected inflation, risk premium, and the ex-ante real rate:

[2] See also a literature survey provided in Stock and Watson (2003).

$$y_t^{(n)} - i_t = \text{slope}_t = \frac{1}{n}\sum_{i=0}^{n-1} E_t r_{t+i} - r_t + \frac{1}{n}\sum_{i=1}^{n} E_t \pi_{t+1} - E_t \pi_{t+1} + rp_t^{(n)}. \tag{4}$$

To implement the decomposition given by equation (4), we assume that the dynamics of both the inflation expectations and the risk premium move on a single factor. We use the median response from the SPF inflation survey denoted by $E_t^{SPF}\pi_{t+1}$ and maintained by the Philadelphia Fed as a proxy for one-year ahead inflation expectations. We measure the variation in bond risk premium using two proxies: the Cochrane-Piazzesi factor (Cochrane and Piazzesi, 2005) denoted by CP_t and a factor developed in Cieslak and Povala (2014a), denoted by $\widehat{cf}_t$.[3] These measures are obtained from predictive regressions of bond excess returns on a set of predictors: a linear combination of forwards for the CP_t factor and a linear combination of yields plus a variable that tracks the long-term inflation expectations in the case of $\widehat{cf}_t$. We decompose the slope by running the following regression:

$$\text{slope}_t = \alpha_0 + \underbrace{\alpha_1}_{\substack{\widehat{cf}_t:\ \ 1.25\ [4.88] \\ CP_t:\ \ 0.16\ [6.68]}} RP_t + \underbrace{\alpha_2}_{\substack{-0.13\ [-1.22] \\ -0.12\ [-1.66]}} E_t^{SPF}\pi_{t+1} + \text{slope}_t^{\perp}. \tag{5}$$

where $RP_t \in \{\widehat{cf}_t, CP_t\}$, the $\bar{R}^2$ is 0.36 for $\widehat{cf}_t$ and 0.50 for CP_t. The residual $\text{slope}_t^{\perp}$ in (5), which we later attribute to the variation in ex-ante real rate, contributes between 50% and 64% of the slope variation. Importantly, in both versions, slope is unrelated to inflation expectations, i.e. taking a difference between the short- and the long-term nominal yield removes completely the inflation expectations from the slope. Using the residual from (5), $\text{slope}_t^{\perp}$, we estimate a predictive regression for the output growth:

$$\Delta g_{t+k/4} = \alpha_0 + \alpha_1 \text{slope}_t^{\perp} + \alpha_2 RP_t + \varepsilon_{t+k/4}. \quad k \in [1, \ldots, 8] \tag{6}$$

Panel B of Table I reports the results for (6) with $\widehat{cf}_t$ as a measure of the risk premium. The results indicate that only $\text{slope}_t^{\perp}$ is a significant predictor of the output growth across all

[3]The availability of risk premium proxies and inflation surveys determines our sample period and its frequency. While the SPF survey is available starting from Q4:1968 at a quarterly frequency, $\widehat{cf}_t$ is available starting from Q1:1972.

horizons. Both the loadings on $\text{slope}_t^{\perp}$ and the $\bar{R}^2$ increase consistently compared to those reported in Panel A. Panel C reports the corresponding results with the Cochrane-Piazzesi factor as a proxy for the risk premium. While the risk premium is not significant up to four quarters horizon, it starts to play a role at longer horizons. Nevertheless, the loadings on the orthogonalized slope are similar in magnitude to those in Panel B with $\widehat{cf}_t$. These results suggest that the link between the slope and output growth is largely a link between the ex-ante real rate and the output growth.[4] The next section introduces a new measure of ex-ante real rate building on the decomposition of the slope discussed above. In Section IV, we give the relationship between the slope and the aggregate output growth an economic interpretation and exploit it to estimate the aggregate EIS.

III. Sources of variation in the slope–a long term perspective

The ex-ante short term real interest rate r_t, which is implicitly defined as $r_t = i_t - \mathrm{E}_t\pi_{t+1}$, is not observable because inflation expectations are not directly observable. A frequently applied approach in the literature is to estimate r_t from the ex-post real rate $i_t - \pi_{t+1}$ by projecting it on the information set available at time t.[5] However, one can obtain biased estimates of the ex-ante real rate if properties of inflation change over time. The ex-ante real rate can also be constructed using inflation expectations from survey data. An attractive feature of this approach is that it requires fewer assumptions on the statistical properties of inflation. However, the inflation survey data are not available in long samples.

[4]Our result is consistent with the slope decomposition performed by Hamilton and Kim (2002). They decompose the slope into expectations hypothesis part and the risk premium, and find that the expectations hypothesis part of the slope is the main driver of the output growth predictability. In our case, the expectations hypothesis part consists of the ex-ante real rate and inflation expectations.

[5]A non-exhaustive list of studies following this approach includes Mishkin (1981); Ferson (1983); Vissing-Jorgensen (2002); Beeler and Campbell (2008); Bansal, Kiku, and Yaron (2012a).

An alternative way to construct a measure for the ex-ante real rate in long samples is to use the long term nominal yield as a proxy for inflation expectations and subtract it from the the one-period nominal yield. We argue that the negative of slope of the term structure is a good proxy for the one-period ex-ante real rate. We denote this measure as $\tilde{r}_t \equiv i_t - y_t^{(n)}$ and set $n = 10$ years given that longer maturities are not available in our sample period 1875-2011. Appendix A provides details on data used to obtain $\tilde{r}_t$.

Compared to other measures of the ex-ante real rate, $\tilde{r}_t$ has several advantages. First, $\tilde{r}_t$ avoids making the linearity assumption which is necessary when applying a linear regression to extract inflation expectations from the realized inflation. Arguably, the linearity assumption is violated in our sample as properties of inflation have changed over time (e.g. Barsky, 1987; Cogley and Sargent, 2014). In other words, $\tilde{r}_t$ assumes the linear relationship between the long-term nominal yield and inflation expectations rather than modeling changing persistence and stochastic volatility of the inflation process. Second, the nominal Treasury yield curve data are available for much longer periods than inflation surveys which allows us to study the ex-ante real rate and its links to key macro variables over long periods.

Whenever possible, we rely on long historical samples. Considering annual data in the sample 1875-2011, we evaluate the links of the slope to ex-ante real rate, risk premium, and inflation. In particular, we provide direct empirical evidence that the slope tracks closely the movements in ex-ante real rate. Before discussing each component individually, Panel A of Table II reports basic statistics for the slope for different sub-samples chosen to reflect the sample periods considered in the literature. Notably, the volatility of the slope is substantial and stable across sub-samples.

III.A. Risk premium

While the risk premium contributes to the slope variation at a quarterly frequency as we show in regression (5), the risk premium plays a minor role in the slope at an annual frequency.

The literature documents that the nominal term spread is a predictor of bond excess returns (Fama and Bliss, 1987; Campbell and Shiller, 1991). Importantly, this evidence is based on the data where the reserve-targeting episode 1979–1982 represents a significant part of the whole sample period.[6] Fama and Bliss (1987) use the data spanning the interval 1964–1985 and Campbell and Shiller (1991) use 1952–1987. Since both the implementation of monetary policy and the yield curve behavior changed in the reserve-targeting period (Clarida, Gali, and Gertler, 2000) this can lead to biased estimates of risk premiums from linear regressions.[7]

We re-evaluate the predictability of bond excess returns by estimating the following predictive regression:

$$rx_{t+1}^{(10)} = \alpha_0 + \alpha_1 \text{slope}_t + \varepsilon_{t+1}, \tag{7}$$

where $rx_t^{(10)}$ is the bond excess return on a nominal ten-year Treasury bond at an annual horizon. In Panel B of Table II, we show that nominal term spread is a good predictor of bond excess returns in periods that include the reserve targeting episode but not before and after. The $\bar{R}^2$ is virtually zero in samples 1875–1979 and 1982–2011, respectively. Additionally, to assess the impact of the reserve-targeting period on bond risk premia, we estimate the following predictive regression:

$$rx_{t+1}^{(10)} = \alpha_0 + \alpha_1 \text{slope}_t + \alpha_2 D_t^{reserv} \times \text{slope}_t + \varepsilon_{t+1}, \tag{8}$$

where the dummy variable D_t^{reserv} indicates the reserve-targeting period. Panel C of Table II reports the results. The dummy regressor is highly statistically significant, increases the $\bar{R}^2$, and weakens the significance of the slope.

[6]Designed to contain surging inflation, on October 6, 1979, the Fed formally announced a change in the conduct of monetary policy with the focus on reserves targeting. As a consequence, this change induced a large increase in the level and volatility of short-term nominal interest rates. In October 1982, the Fed abandoned the formal M1 growth target.

[7]Indeed, Evans and Lewis (1994) and Lewis (1991) provide a "peso problem" explanation for the predictability of bond excess returns in this particular period. They show that, in the presence of anticipated shifts in the term structure of interest rates, estimates of risk premiums from predictive regressions are upwardly biased.

III.B. Inflation expectations

If the slope contains information about future inflation it should predict inflation changes. To assess this, we estimate the following regression:

$$\pi_{t+h+1} - \pi_{t+1} = \alpha_0 + \alpha_1 \text{slope}_t + \varepsilon_{t+h+1}, \tag{9}$$

where π_{t+h+1} is the realized inflation between time t and $t+h+1$, $h > 1$ is given in years and π_{t+1} represents the realized inflation one year ahead. Table III reports the regression results for different sub-samples. Across horizons, the predictability is contained to periods centered around 1979–1982. Neither in the full sample 1875–2011 nor in sub-samples that exclude the reserve-targeting period are inflation changes predictable by the slope. The results indicate that the nominal term spread is unrelated to future changes in inflation. While the previous literature documents that the slope predicts inflation changes (Fama, 1990; Mishkin, 1990a,b), we argue that, similar to the evidence on bond excess returns, the predictability of inflation changes is driven by the reserve-targeting episode.

III.C. Ex-ante real rate

We provide three empirical facts to support our claim that $\tilde{r}_t$ is a good proxy for the variation in the ex-ante real rate. First, we use it as a single instrument to back out the ex-ante real rate from the ex-post real rate:

$$i_t - \pi_{t+1} = \underbrace{\alpha_0}_{0.021\ [3.50]} + \underbrace{\alpha_1}_{1.14\ [5.01]} \tilde{r}_t + \varepsilon_{t+1} \quad \bar{R}^2 = 0.18, \tag{10}$$

where the t-statistics reported in parentheses are Newey-West-adjusted with four lags and the sample period is 1875-2011. The estimation results show that $\tilde{r}_t$ is strongly related to the ex-post real rate with the correct sign and the null $\alpha_1 = 1$ cannot be rejected. The intercept in regression (10) differs significantly from zero which indicates that $\tilde{r}_t$ captures the dynamics of the ex-ante real rate but not its average level. For comparison, we also estimate the ex-ante real rate using the yield on the nominal three-month T-bill and the past inflation rate as instruments as has been done in the literature (e.g. Hall, 1988; Yogo, 2004; Constantinides and Ghosh, 2011; Beeler and Campbell, 2012; Bansal, Kiku, and Yaron, 2012b):

$$i_t - \pi_{t+1} = \underbrace{\alpha_0}_{-0.015\ [-3.06]} + \underbrace{\alpha_1}_{1.13\ [10.53]} i_t + \underbrace{\alpha_2}_{-0.65\ [-7.14]} \pi_t + \varepsilon_{t+1} \quad \bar{R}^2 = 0.52. \tag{11}$$

$$i_t - \pi_{t+1} = \underbrace{\alpha_0}_{-0.01\ [-1.06]} + \underbrace{\alpha_1}_{0.98\ [7.55]} i_t + \underbrace{\alpha_2}_{-0.59\ [-5.53]} \pi_t + \underbrace{\alpha_3}_{0.003\ [1.39]} \tilde{r}_t \tag{12}$$

$$+ \underbrace{\alpha_4}_{0.001\ [0.30]} \tilde{r}_{t-1} + \underbrace{\alpha_5}_{-0.14\ [-1.46]} \Delta c_{t-1} + \varepsilon_{t+1} \quad \bar{R}^2 = 0.53, \tag{13}$$

Figure 1 plots these two versions of the ex-ante real rate. The figure shows that including past inflation significantly improves the statistical fit, in fact the $\bar{R}^2$ increases from 0.18 obtained in regression (10) to 0.52 in regression (11). A closer look reveals that most of the improvement comes from fitting extreme values of inflation during the two World Wars and the Great Depression. Thus, the specification given by (11) generates a measure of ex-ante real rate that is too volatile: the standard deviation of the estimated ex-ante real rate in the period 1915-1950 is 5.3% while the standard deviation of nominal three-month T-bill, which comprises both the ex-ante real rate and inflation expectations, is 2.2%, i.e. less than half in the same period. Consequently, it does not seem plausible that so much variation in the ex-post real rate has been anticipated by investors at an annual horizon.[8] Notably, the two estimates of the ex-ante real rate diverge during the post-Volcker period where the estimate obtained from (11) inherits a downward trend which is not present in the ex-ante real rate estimated with $\tilde{r}_t$.

[8]Using more recent survey data for the Federal funds rate (1982-2010), Cieslak and Povala (2014b) show that investors anticipate only around 20% of changes in the Federal funds rate at an annual horizon.

Second, for the slope to be a good measure of real rate variation, the term structure of inflation expectations should be flat, i.e. move on one persistent factor. In such a case, we should observe that the slopes of the real and nominal term structures are highly correlated. We use the long-term survey data from the Blue Chip Economic Indicators (BCEI) panel for the 1984–2010 period to assess these implications. Panel a of Figure 2 shows the term structure of inflation expectations and confirms that it is well described by a single level factor. Panel b of Figure 2 plots the real term spread[9] superimposed with the nominal term spread for the period 1979–2010. Their correlation is 0.84.

Finally, Figure 3 compares $\tilde{r}_t$ to the ex-ante real rate obtained using one year ahead inflation forecast from the SPF survey denoted by r_t^{surv}.[10] Our measure of real rate variation $\tilde{r}_t$ closely tracks the movements in r_t^{surv}. The only notable divergence is due to the downward trend present in the survey-based real rate in the post-Volcker period (1982-2013). The presence of the trend component in the ex-ante real rate has been documented in the literature (e.g. Neely and Rapach, 2008). Interestingly, the downward trend in the real rate follows an abrupt increase in the real rate during the reserve-targeting period (e.g. Blanchard and Summers, 1984). The real rate trend is also highly positively correlated with the trend in inflation in the post-Volcker period. These observations are consistent with an interpretation of the real rate trend induced by the monetary policy designed to contain high inflation. We need the stationary part of the ex-ante real to estimate the EIS from an aggregate Euler equation. By removing the trend component of the real rate, $\tilde{r}_t$ captures the stationary part of ex-ante real rate which is our object of interest.

[9] We construct the real term spread by subtracting the corresponding survey based inflation forecasts from the three-month and ten-year nominal yields.

[10] Similar results are obtained with the Livingston survey on inflation which goes back to 1955 and is conducted semi-annually.

IV. Estimating the aggregate Euler equation

Intertemporal Euler equation is at the core of a broad range of asset pricing and macro models. For a variety of utility functions, including the Epstein-Zin specification, and under the assumption of joint log-normality of consumption growth and asset returns (e.g. Hansen and Singleton, 1983; Hall, 1988; Attanasio and Weber, 1989), its linearized form is given by:

$$\mathrm{E}_t \Delta c_{t+1} = \mu + \sigma \mathrm{E}_t \log\left(1 + R_{rf,t+1}\right), \tag{14}$$

where Δc_{t+1} is the log consumption growth, σ is interpreted as the intertemporal elasticity of substitution (EIS), and $R_{rf,t+1}$ is the ex-post real interest rate. The constant term is a function of the variances and covariances of Δc_{t+1}, $\log\left(1 + R_{rf,t+1}\right)$, and the preference parameters. As rational investors would take advantage of higher interest rates and postpone consumption into next period σ should be positive. Equation (14) holds for each agent individually. A representative-agent assumption is needed to obtain an unbiased estimate of σ from the aggregate consumption (output) data. To the extent that the average consumer is different from the average investor, estimates of the EIS are biased (Guvenen, 2006).

IV.A. Estimation

We estimate the log-linearized Euler equation (14) via instrumental variables which we implement as a two-stage least squares. The key for obtaining our results is the selection of instruments for the ex-post real rate R_{rf_t+1}. Equations (10)–(11) indicate that $\tilde{r}_t$ is a valid instrument for the ex-ante real rate while adding past inflation to the instrument set leads to over-fitting of the ex-ante real rate. We start by estimating (14) with $\tilde{r}_t$ as the only instrument for $\mathrm{E}_t \log\left(1 + R_{rf,t+1}\right)$. We contrast $\tilde{r}_t$ with the instrument set commonly used in the literature and assess the impact of instrument selection on the magnitude of EIS estimates. The second instrument set contains $\tilde{r}_t$, its lagged value, three-month T-bill, lagged consumption (output) growth, and past inflation.

We consider the sample period 1875-2009 at an annual frequency. We use the US real per capita annual data on total consumption and output for the sample period 1875–2009 from the data set constructed by Barro and Ursua (2010).[11] More details regarding the data sets and their sources are provided in Appendix A. The output data allow us to evaluate the aggregate output Euler equation (IS equation) relevant for the New-Keynesian models. Figure 4 plots the real consumption growth Δc_{t+1} (Panel a) and output growth Δy_{t+1} (Panel b) superimposed with $\tilde{r}_t$ for the period 1875-2009. The unconditional correlation between $\Delta c_{t+1}, \Delta y_{t+1}$ and $\tilde{r}_t$ is -0.34 and -0.30, respectively. Table IV reports the estimates of σ using two different instrument sets obtained from the annual data for the period 1875–2009. Panel A reports the results for the consumption growth and Panel B for the real output growth. The most important finding is that, using $\tilde{r}_t$ as a single instrument for the ex-ante real rate, the estimates of the aggregate EIS are negative around -0.5 and with the 95% confidence interval excluding zero and positive parameter values. Extending the instrument set by the three-month T-bill, lagged consumption (output) growth, lagged $\tilde{r}_t$, and past inflation leads to estimates of the EIS that are close to zero and statistically insignificant—a result consistent with the literature, e.g. Hall (1988); Campbell (2003); Yogo (2004). We argue that the over-fitting the ex-ante real rate in periods with volatile inflation leads to insignificant estimates of the EIS. The test rejects the null of weak instruments for both instrument sets, confirming $\tilde{r}_t$ as a suitable instrument for the ex-ante real rate.

IV.B. Robustness

Quarterly data

Barro and Ursua's data set does not distinguish between non-durable and durable consumption expenditures due to limitations on data availability. Our previous results are

[11] This dataset has been recently used in several papers, e.g. Nakamura, Steinsson, Barro, and Ursua (2013); Rangvid, Santa-Clara, and Schmeling (2014).

obtained with total consumption expenditures which include durable expenditures that flow into consumption over several periods. Hence, including durable expenditure can introduce a bias into EIS estimates. Given that expenditures on durable goods are roughly 15% of the non-durable consumption and services, the potential bias in the EIS estimates is likely to be small.[12] We address this concern by estimating the Euler equation distinguishing between durable and non-durable expenditures. We use quarterly data in the sample Q1:1960–Q4:2013.[13] The quarterly real per capital data on consumption growth as well as the output growth data are constructed from the Bureau of Economic Analysis tables.

Table V reports the results: Panel A displays the estimates of EIS using expenditures on non-durable goods and services. Panel B shows the results obtained with durable goods expenditures, and Panel C reports the EIS estimates using total consumption. Panel D shows the estimates for output growth. The EIS estimate for the non-durable consumption growth using $\tilde{r}_t$ as an instrument is -0.42 with a standard error of 0.24 which confirms the results from the long sample (-0.44 with a standard error of 0.15). The EIS estimate for the durable consumption growth is -3.97 with a standard error of 1.58 and -0.82 with a standard error of 0.40 for the output growth. Not surprisingly, the EIS estimate is significantly higher for durable growth expenditures as compared to the non-durable consumption growth. Thus when the total consumption growth is used, the presence of durable goods expenditures slightly increases the magnitude of EIS estimates. However, the difference is well within one standard deviation of the EIS estimate from the non-durable consumption. There seems to be no evidence that the negative estimate of the aggregate EIS is driven solely by the non-durable consumption expenditures or a particular sample period. The weak instrument test for $\tilde{r}_t$ rejects the null of weak instruments. Comparing the instrument sets in Table V shows that including additional instruments such as past inflation leads to an estimate of the ex-ante real rate that contains the trend component and, as a result, EIS estimates that

[12]The main motivation for excluding expenditures on durables is to minimize the measurement error originating from the discrepancy between expenditures and the consumption itself. However, with decreasing measurement frequency the error becomes smaller.

[13]Several recent macro studies such as Fuhrer and Rudebusch (2004); Canzoneri, Cumby, and Diba (2007); Jurado, Ludvigson, and Ng (2014) start their sample in 1960s.

are statistically insignificant. As such, the success of instrumenting by $\tilde{r}_t$ lies in avoiding the over-fitting the ex-ante real rate.

For comparison, the estimate of the aggregate EIS for the aggregate output growth using the survey-based ex-ante real rate is -0.22 with a standard error of 0.14. The lower magnitude and weaker significance are due to the trend component that is present in the survey-based ex-ante real rate but not in $\tilde{r}_t$.

Household decision interval

By considering quarterly data frequency, we also address the concern related to the household's decision interval. If the decision interval is shorter than one year, the aggregation introduces a spurious autocorrelation in Δc_{t+1} and the estimates of the EIS are biased. The main argument for measuring the consumption and output at annual rather than higher frequencies are the transaction costs and other frictions which might prevent households from instantaneous consumption smoothing (Gabaix and Laibson, 2001; Jagannathan and Wang, 2007). Additionally, the variation in real interest rates is relatively persistent (the annual AR(1) coefficient is 0.68), hence the consumption could react slowly to changes in the real interest rate because the utility loss induced by the slow adjustment is low.

International evidence

It is possible that negative relationship between the ex-ante real rate and the aggregate consumption growth is specific to the US data. To evaluate this possibility, Table VI reports the EIS estimates for selected countries with developed bond markets. The estimates are consistently negative and have, in some cases, a larger magnitude than in the US. Arguably, each of these countries follow different monetary policies. Therefore, the evidence from a panel of countries suggests that negative link between the ex-ante real rate and the consumption growth is not driven by monetary policy or country-specific regulatory changes.

Econometric issues

Estimating the EIS from the log-linearized aggregate Euler equation is subject to a number of potential biases. One obvious source of the bias is the stochastic volatility of consumption growth. Attanasio and Low (2004) show that, even in the presence of stochastic volatility, the log-linearized Euler equation yields unbiased estimates of the EIS if the sample period is long enough, which is the case in this paper. Furthermore, stochastic volatility is less of a concern in aggregate data where the idiosyncratic volatility is averaged out. Beeler and Campbell (2008) show that the bias due to stochastic volatility does not play an important role, at least for the volatility process calibrated to the US aggregate consumption data. We empirically evaluate the impact of stochastic volatility on estimates of the aggregate EIS by estimating an augmented version of equation (14):

$$\Delta c_{t+1} = \mu + \underbrace{\sigma}_{-0.46\ [-2.89]} E_t \log(1 + R_{f,t+1}) + \underbrace{\delta}_{0.003\ [0.73]} \hat{v}_t^2 + \varepsilon_{t+1}, \tag{15}$$

where $\hat{v}_t^2$ is an empirical proxy for the consumption volatility. Following Andersen, Bollerslev, and Diebold (2003); Bansal, Khatchatrian, and Yaron (2005), it is constructed as: $\hat{v}_t^2 = \log \sum_{i=1}^{5} |\eta_{t-i}|$ where η_t is the innovation in consumption growth obtained from an AR(1) estimated on Δc_t recursively using the window size of 30 years. Equation (15) is implemented as a two-stage least squares using $\tilde{r}_t$ as an instrument for $E_t \log(1+R_{f,t+1})$. Robust z-statistics are reported in parentheses. The results show that the volatility of consumption growth is not statistically significantly related to future consumption growth. Moreover, the presence of stochastic volatility does not alter the negative relationship between the ex-ante real rate and the consumption growth.

It can be that our results are driven by a few outliers and the negative relationship between consumption growth and the real rate disappears once we smooth out the outliers. To address this concern, we construct moving averages of both quantities, demeaned Δc_{t+1} and $\tilde{r}_t$, and run a univariate regression with filtered series. To compute moving averages, we use the filter proposed in Lucas (1980). For a covariance stationary series x_t, it is given by:

$$\bar{x}_t(\beta) = \alpha \sum_{k=-n}^{n} \beta^{|k|} x_{t+k} \tag{16}$$

$$\alpha = \frac{(1-\beta)^2}{1-\beta^2-2\beta^{n+1}(1-\beta)}, \tag{17}$$

where $\beta \in [\,0, 1\,)$. The goal is to evaluate the relationship between consumption growth and $\tilde{r}_t$ across frequencies. The measure of the ex-ante real rate is persistent and it should be reflected in the persistent variation of expected consumption growth. We study the following four degrees of smoothing $\beta \in \{0, 0.8, 0.95, 0.98\}$.[14]

Figure 5 displays the comovement of consumption growth with $\tilde{r}_t$ at different frequencies as measured by values of β. The negative relationship between $\tilde{r}_t$ and consumption growth is present at all frequencies. Moreover, the regression coefficient, which is given by the slope of the dash-dotted line, shows a remarkable stability for all degrees of smoothing. The estimated slope coefficients are close to the EIS estimates reported in Panel A of Table IV.

Alternative explanations

The relationship between the ex-ante real interest rate and consumption growth in equation (14) might appear negative even though the true relationship is positive. This is possible if μ in equation (14) is time-varying and strongly negatively correlated with the ex-ante real rate.[15] In such a case, one obtains a negative estimate of σ even though the true EIS is positive. However, for this to happen, the time discounting shock would have to be more volatile than the ex-ante real rate, which, together with the highly negative correlation seems implausible. Alternatively, it can be that the unobserved returns to human capital are negatively correlated with the real short rate. However, Lustig, Van Nieuwerburgh, and Verdelhan (2013) show that returns on human wealth are positively related to real bond returns. Therefore, the negative relationship between consumption growth and the real

[14] Note that $\beta = 0$ corresponds to a univariate regression of unfiltered series. For $\beta \to 1$, the regression approximately measures the relationship at frequency $\omega = 0$ in the frequency domain, for details see Whiteman (1984).

[15] The discount factor shock is often introduced in the empirical implementations of dynamic stochastic general equilibrium models (Justiniano and Primiceri, 2008). The factor is usually interpreted as a shock to aggregate demand or demographics, see also Albuquerque, Eichenbaum, and Rebelo (2014).

interest rate cannot be replicated for positive values of σ at the aggregate level unless one is willing to accept counterfactual assumptions.

In sum, the EIS estimates are consistently negative with $\tilde{r}_t$ as an instrument and thus counter the standard economic intuition which asserts that $\sigma > 0$, i.e. a positive shock to the real interest rate induces agents to postpone today's consumption to the next period.[16] Consequently, one cannot interpret estimates of σ as a structural parameter. Overall, negative estimates of σ indicate the need for a modelling framework that allows for a substantial heterogeneity of agents in the economy with respect to their consumption and wealth.

V. Model

This section outlines a general equilibrium model sticky prices and limited asset market participation. The model replicates both the negative sign and the magnitude of the relationship between the ex-ante real rate and the aggregate consumption/output growth. The model setup is closely related to Gali, Lopez-Salido, and Valles (2004) and Bilbiie (2008). While these papers focus on effects of monetary policy and equilibria determinacy in the presence of limited market participation, we evaluate the asset pricing implications of limited participation. The key difference to other limited participation models put forward in the literature, such as Guvenen (2006), is that these models cannot generate the negative relationship between the ex-ante real rate and the aggregate consumption growth.[17] To make the key mechanism transparent, we restrict the technology to be the only economic

[16]Note that Hansen and Singleton (1984, 1996) in some cases obtain negative but insignificant estimates of the risk aversion parameter. Hall (1988) also estimates a negative value for σ using annual data in the sample spanning 1924-1940 and 1950-1983. Similarly, Yogo (2004) reports negative albeit in most cases insignificant estimates of EIS for a panel of developed countries. Using the US data on the long sample period (1891-1997), Campbell (2003) estimates a negative though insignificant EIS parameter. These studies interpret negative estimates of EIS as implausible.

[17]Guvenen (2006) studies a model with limited stock market participation in which two types of agents have different elasticities of intertemporal substitution. Importantly, agents that do not participate in the stock market can trade bonds and thus perform intertemporal smoothing.

disturbance in the model. As such, the model is too simple to be evaluated on quantitative predictions along other dimensions.

The economy consists of two types of households, a representative final-good-producing firm, a continuum of intermediate-goods-producing firms and a central bank.

Households

We distinguish between two types of households: *savers* denoted by subscript s and *rule-of-thumb* households denoted by the subscript r. There is a continuum of households on [0,1] where $1-\lambda$ fraction of them are savers who participate in asset markets to smooth consumption and own all assets in the economy. The rule-of-thumb agents on the interval $[0,\lambda]$ do not participate in financial markets to smooth their consumption and do not have any wealth. Both types of households have an identical period utility function U:

$$U\left(C_{i,t},N_{i,t}\right)=\frac{C_{i,t}^{1-\gamma}}{1-\gamma}-\frac{N_{i,t}^{1+\eta}}{1+\eta}, \tag{18}$$

where $i\in\{s,r\}$.[18] The parameter $\gamma>0$ represents the degree of risk aversion, $\eta>0$ is the inverse of labor supply elasticity. Savers solve the usual dynamic optimization problem:

$$\max \mathrm{E}_t\sum_{j=0}^{\infty}\beta^j U\left(C_{s,t+j},N_{s,t+j}\right), \quad \beta\in(0,1) \tag{19}$$

by choosing the consumption $C_{s,t}$ and worked hours $N_{s,t}$ every period subject to the budget constraint expressed in nominal terms:

$$B_{s,t+1}+\Theta_{s,t+1}V_t\leq B_{s,t}R_t+\Theta_{s,t}\left(V_t+P_tD_t\right)+W_tN_{s,t}-P_tC_{s,t}. \tag{20}$$

[18]Separable preferences are used for simplicity. The utility function given by (18) is not compatible with a balanced growth path. To resolve this, one could introduce a non-separable utility specification. The following specification is common in the RBC literature: $U\left(C_t,N_t\right)=\frac{1}{1-\sigma}\left[\left(C_t v\left(N_t\right)\right)^{1-\sigma}-1\right]$. The main result is obtained also with this specification.

$B_{s,t}$ is the quantity of one-period nominal bonds at the beginning of period t,[19] $\Theta_{s,t}$ denotes the holdings of savers in firms,[20] V_t is the market value of firms and D_t are real dividends, W_t denotes the nominal wage, P_t is the price level and R_t is the one-period nominal interest rate. The no-arbitrage condition implies the existence of a nominal stochastic discount factor $M_{t,t+1}$ that prices all uncertain future cash flows:

$$V_t = \mathrm{E}_t \left[M_{t,t+1} \left(V_{t+1} + P_{t+1} D_{t+1} \right) \right], \tag{21}$$

and bonds. The stochastic discount factor together with the assumption of frictionless financial markets (for savers) allows to rewrite (20) in terms of present values:

$$\mathrm{E}_t \sum_{j=t}^{\infty} M_{t,t+j} P_j C_{s,j} \leq V_t + \mathrm{E}_t \sum_{j=t}^{\infty} M_{t,t+j} W_j N_{s,j}. \tag{22}$$

The remaining first order conditions for savers are:

$$\beta \frac{U_C \left(C_{s,t+1} \right)}{U_C \left(C_{s,t} \right)} = M_{t,t+1} \frac{P_{t+1}}{P_t} \tag{23}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \left[\left(\frac{C_{s,t+1}}{C_{s,t}} \right)^{-\gamma} \frac{P_t}{P_{t+1}} \right] \tag{24}$$

$$\frac{N_{s,t}^{\eta}}{C_{s,t}^{-\gamma}} = \frac{W_t}{P_t}. \tag{25}$$

The optimization problem of rule-of-thumb consumers involves only the intratemporal trade-off:

$$\max_{C_{r,t}, N_{r,t}} \quad \frac{C_{r,t}^{1-\gamma}}{1-\gamma} - \frac{N_{r,t}^{1+\eta}}{1+\eta}, \tag{26}$$

[19] Nominal bonds are in the zero net supply because markets are complete and savers are homogenous. Hence, all savers in the economy can be replaced by a representative agent.

[20] In a model with the representative agents $\Theta = 1$. In this model, it will depend on the fraction of savers in the whole population: $\Theta = \frac{1}{1-\lambda}$.

subject to the budget restriction $C_{r,t}P_t = W_tN_{r,t}$. The first order condition for the rule-of-thumb household is:

$$\frac{N_{r,t}^{\eta}}{C_{r,t}^{-\gamma}} = \frac{W_t}{P_t}. \tag{27}$$

Production

The production of final good Y_t is described by a CES production function with elasticity ε, which aggregates the intermediate goods indexed by k, i.e. $Y_t = \left(\int_0^1 Y_t\,(k)^{(\varepsilon-1)/\varepsilon}\,dk\right)^{\varepsilon/(\varepsilon-1)}$. Firms producing intermediate goods are characterized by a linear production technology without capital:

$$Y_t(k) = A_tN_t(k) - F \text{ if } N_t(k) > F \text{ and } 0 \text{ otherwise}, \tag{28}$$

where F denotes the fixed costs and A_t represents the technology, where $a_t = \log A_t$ is assumed to evolve as:[21]

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a, \tag{29}$$

with $0 \leq \rho_a \leq 1$ and $\varepsilon_t^a \sim N\left(0, \sigma_a^2\right)$. Firms producing intermediate goods face a downward-sloping demand curve: $C_t(k) = \left(\frac{P_t(k)}{P_t}\right)^{-\varepsilon} C_t$. They take wages as given and the cost minimization implies the nominal marginal cost: $MC_t = W_t/A_t$. The total profit D_t is given by $D_t = \left(1 - \left(MC_t/P_t\right)\Delta_t\right)Y_t$, where $\Delta_t = \int_0^1 \left(P_t(k)/P_t\right)^{-\varepsilon} dk$ is defined as relative price dispersion. Savers, who hold shares in firms, maximize their total value by choosing price $P_t(k)$. Nominal rigidities are modeled via Calvo staggered pricing with ω representing the fraction of firms being unable to adjust their prices. The optimal price is:

$$P_t^{opt}(k) = \mathrm{E}_t \sum_{j=0}^{\infty} \frac{\omega^j M_{t,t+j} P_{t+j}^{\varepsilon-1} Y_{t+j}}{\mathrm{E}_t \sum_{m=0}^{\infty} \omega^m M_{t,t+m} P_{t+m}^{\varepsilon-1} Y_{t+m}} MC_{t+j}. \tag{30}$$

[21] The technology shock can have a unit-root, in which case the output, consumption, and real wages would need to be stochastically detrended to obtain stationary dynamics.

The aggregate price index evolves as:

$$P_t^{1-\varepsilon} = (1-\omega)\left(P_t^{opt}\right)^{1-\varepsilon} + \omega P_{t-1}^{1-\varepsilon}. \tag{31}$$

The Philips curve follows from linearized versions of (30) and (31), (e.g. Woodford, 2003).

Equilibrium

The equilibrium requires that all markets clear: labor markets clearing implies $N_t = \lambda N_{r,t} + (1-\lambda) N_{s,t}$. Since we abstract from investment and government, goods market clearing delivers $C_t = Y_t$ and $C_t = \lambda C_{r,t} + (1-\lambda) C_{s,t}$. Asset market clearing implies that $\Theta_{s,t} = \frac{1}{1-\lambda}$ for all t. Bonds are in zero net supply.

V.A. Key model equations

The aggregate dynamics are described by the following three equations (derivations and the discussion of steady states are provided in Appendix E). Lower case letters below denote the log-deviation of a given variable from its steady state value.

Aggregate Euler equation. The Euler equation of savers has the usual form:

$$\mathrm{E}_t c_{s,t+1} - c_{s,t} = \frac{1}{\gamma}\left(i_t - \mathrm{E}_t \pi_{t+1}\right). \tag{32}$$

The output Euler equation is obtained by aggregating the consumption of savers and rule-of-thumb agents:

$$\mathrm{E}_t y_{t+1} - y_t \quad = \tfrac{1}{\delta\gamma}\left(i_t - \mathrm{E}_t \pi_{t+1}\right) + \chi\left[\mathrm{E}_t a_{t+1} - a_t\right], \tag{33}$$

where y_t represents the log of aggregate output, $\pi_t = \log P_t/P_{t-1}$ is the inflation and $\chi = -\frac{1}{\frac{\kappa\lambda\eta+1-\lambda}{\lambda\eta(1+\kappa)} - \frac{1}{1+\mu}}$. The magnitude of $\frac{1}{\delta\gamma}$ crucially depends on δ, which is a function of structural parameters:

$$\delta = \frac{(\kappa\lambda\eta + 1 - \lambda)(1+\mu) - \lambda\eta(1+\kappa)}{(1-\lambda)(1+\mu)[\kappa\lambda\eta + \gamma\lambda(1+\kappa) + 1 - \lambda]},$$

where $\kappa \equiv \frac{1-\gamma}{\eta+\gamma}$. In case of full market participation ($\lambda = 0$) $\delta = 1$, and the standard IS curve is recovered. Importantly, δ can have positive or negative sign depending on the combination of structural parameters in the numerator. Equation (33) illustrates how negative sign in the relationship between the ex-ante real rate and the consumption growth arises if $\delta < 0$ even though the EIS of savers given by $1/\gamma$ is positive.

Phillips curve. Nominal rigidities introduce a trade-off between the output gap $x_t = y_t - y_t^*$ and inflation:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \psi\vartheta\,(y_t - y_t^*)\,. \tag{34}$$

Natural level of output, y_t^*, which depends only on the exogenous technology shock a_t, is achieved when prices are flexible, i.e. $\omega = 0$ (see Appendix E.3 for the derivation). Unlike most of the recent New-Keynesian model specifications, both the aggregate output equation and the Phillips curve are purely forward-looking. In our setup, the Phillips curve is not influenced by limited participation.

Monetary policy. The central bank sets the policy rate according to the following interest rate rule:

$$i_t = \phi_i i_{t-1} + (1-\phi_i)\left[\phi_\pi \pi_t + \phi_x\,(y_t - y_t^*)\right]. \tag{35}$$

Consistent with the empirical evidence, we assume that the central bank moves the policy rate gradually, which introduces a substantial degree of smoothing captured by ϕ_i.

V.B. Model calibration

Where possible, the choice of parameter values is guided by microeconomic evidence. We calibrate the model at a quarterly frequency.

Elasticity of labor supply. Chetty, Guren, Manoli, and Weber (2011) argue that the Frisch elasticity of labor supply should be set to 0.75 which implies $\eta = 1.333$. However, to assess the sensitivity of our results, we study a range of values reported in the literature, $\eta \in \{1, 1.333, 2, 4\}$.

Asset market participation. We set the fraction of non-participating households $\lambda = 0.7$ which is motivated by the evidence from Vissing-Jorgensen (2002) who classifies 21.75 percent of households as stockholders and 31.40 percent as bondholders, based on the Consumer Expenditure Survey (CEX) for the period 1980-1996. Similarly, Guvenen (2006) argues that $\lambda = 0.7$ represents a lower bound for the fraction of non-participating households. One concern with selecting a value for λ motivated by historical data is the fact that stock market participation of households has been increasing over time. We provide a detailed discussion of this issue in Appendix D. To assess the impact of λ on our results, we vary its values between 0 and 1.

Rigidities. The elasticity of substitution among intermediate goods is set to $\varepsilon = 6$ which implies the steady-state markup value $\mu = 0.2$ (Eichenbaum and Fisher, 2007). This parameter has a small effect on δ for any plausible markup value. The Calvo parameter of $\omega = 0.75$ implies that firms can reset prices once a year.

Risk aversion. We set $\gamma = 2$ but study parameter values ranging from 0.5 through 10. Values of γ that are substantially higher than unity are commonly used in the asset pricing literature. In contrast, Chetty (2006) obtains an upper bound for γ equal to two by exploiting the labor supply behavior.

Monetary policy. The smoothing parameter ϕ_i is set to 0.85 which is motivated by Coibion and Gorodnichenko (2012). We set $\phi_\pi = 0.4$ and $\phi_x = 1.2$. In the standard New-Keynesian model without limited participation, coefficients above one for ϕ_π are necessary to ensure the determinacy (Taylor rule). However, for a sufficiently high degree of limited market participation, we have that $0 < \phi_\pi < 1$ leads to determinate equilibria as discussed in (Gali, Lopez-Salido, and Valles, 2004; Bilbiie, 2008).[22]

Technology shock. The single exogenous shock of the model is the technology shock. Guided by Fernald (2012), we set $\rho^a = 0.3$ and $\sigma_a = 0.01$.

All baseline parameters are collected in Table VII.

V.C. Model solution

Equations (33)–(35) together with the exogenous technology shock determine the law of motion of the endogenous variables in the model, which is given by:

$$z_t = \Gamma z_{t-1} + \Xi a_t, \tag{36}$$

where $z_t = [\pi_t, y_t, i_t, y_t^*]'$. More details about the model solution are provided in Appendix E.4. Numerically solving the model for these parameter values gives the following dynamics:

$$\Gamma = \begin{bmatrix} 0 & 0 & 0.9570 & 0 \\ 0 & 0 & 0.2224 & 0 \\ 0 & 0 & 0.9475 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Xi = \begin{bmatrix} 0.3335 \\ 1.2423 \\ 0.1087 \\ 0.7496 \end{bmatrix}. \tag{37}$$

[22] In this model, π_t is a deviation of inflation from its steady state. Post-war US inflation has a stochastic trend, and Taylor rule estimates that explicitly account for it, indicate that the central bank reacted by more than one-for-one to the inflation trend, but the coefficient on the deviations from the stochastic trend is indeed less than unity. Here, we abstract from the stochastic inflation target. See Cogley and Sbordone (2008) and Coibion and Gorodnichenko (2011) for the analysis of the New Keynesian models with inflation trend.

The single most important observation about (37) is that the dynamics of both the inflation π_t and the nominal short rate i_t are persistent. The persistence in inflation is driven by the inverted sign in the aggregate output Euler equation. This result is surprising given that the Phillips curve and the aggregate Euler equation are purely forward-looking. A full participation model does not generate such a persistence without adding backward-looking inflation indexation in the Phillips curve (Christiano, Eichenbaum, and Evans, 2005; Cogley and Sbordone, 2008).

VI. Model implications

This section analyzes the key mechanism in the model that leads to the negative sign in the relationship between the aggregate consumption (output) growth and the ex-ante real rate. Subsequently, it discusses the link between the aggregate consumption and the consumption of savers. Finally, it shows how the negative sign of the aggregate Euler equation helps reconcile the consumption-based models with the data.

VI.A. Justifying $\tilde{r}_t$ as a proxy for the real rate

We simulate the model at the baseline parameters and, following Bekaert, Cho, and Moreno (2010), we construct a term structure of nominal interest rates.[23] The details of the New-Keynesian term structure model and of the simulation exercise are provided in Appendix E.5. Figure 6 shows the histogram of correlations between $\tilde{r}_t$ and $i_t - \mathrm{E}_t\pi_{t+1}$. The simulation shows that $\tilde{r}_t$, which is the negative of the slope of the simulated term structure, closely tracks the real rate constructed as $i_t - \mathrm{E}_t\pi_{t+1}$, where a model-consistent process for realized inflation is used to compute $\mathrm{E}_t\pi_{t+1}$. Median correlation is around 0.7. The law of motion of endogenous variables given by (37) indicates that the inflation is more persistent than the

[23] In the model by Bekaert, Cho, and Moreno (2010) the risk premium is constant. Given that we focus on intertemporal trade-off and expectations, time-varying risk premia are of secondary importance.

output. Consequently, the long term nominal yield is driven mainly by inflation expectations and the nominal term spread captures an important part of variation in the real rate.

VI.B. What makes the sign in the aggregate Euler equation negative?

Figure 7 plots the values of the aggregate EIS given by $\frac{1}{\delta\gamma}$ for a range of structural parameters: risk aversion γ, degree of limited asset market participation λ, and elasticity of labor supply η together with its estimate from the consumption data in the period 1875-2009 (Panel A of Table IV). The figure shows that parameter values for λ, γ, and η motivated above are consistent with the empirical estimate of the relationship between the aggregate consumption growth and the ex-ante real rate as measured by $\frac{1}{\delta\gamma}$. Conditional on a moderate degree of risk aversion, $\gamma = 2$, Figure 8 illustrates the sensitivity of $\frac{1}{\delta\gamma}$ to two key ingredients that drive the sign of δ: inelastic labor supply as measured by η and limited market participation. δ changes its sign at the threshold value of market participation denoted by $\hat{\lambda}$ and derived in Appendix E.2.[24] All considered parameter values for labor supply elasticities imply that the sign of δ switches for the degree of non-participation between 0.25 and 0.55, i.e. well below the levels indicated by the empirical evidence.

The intuition for the negative relationship between the real rate and expected aggregate consumption growth is as follows. A positive exogenous technology shock increases the natural level of output y_t^* and thus changes the output gap x_t. Depending on the configuration of the parameter values, in particular those in the monetary policy reaction function, the increase in y_t^* translates through the monetary policy reaction into a positive or negative shock to the real interest rate. In the full participation case, a positive shock to the real interest rate reduces the demand today as it induces the agents to seize better investment opportunities

[24]The aggregate EIS has a discontinuity point at $\hat{\lambda}$. For this reason, the negative sign of the EIS cannot be achieved by introducing agents with different non-zero EIS as is done in (Guvenen, 2006). If all agents can trade at least in the bond market and thus smooth consumption intertemporally, it is impossible to obtain the negative sign in $\frac{1}{\delta\gamma}$.

and substitute today's consumption into the future. In the limited participation model, only savers react to the real rate shock and substitute consumption intertemporally. Intertemporal choices of savers shift the labor supply curve (derived in Appendix E.2). An increase in the consumption of savers leads to a higher wage:

$$w_t = \frac{\eta}{\eta\lambda\kappa + (1-\lambda)} n_t + \frac{(1-\lambda)\gamma}{\eta\lambda\kappa + (1-\lambda)} c_{s,t}. \tag{38}$$

For the parameter values given in Table VII, $\frac{(1-\lambda)\gamma}{\eta\lambda\kappa+(1-\lambda)} > 0$, i.e. a positive shock to the real interest rate reduces $c_{s,t}$ and thus the real wage w_t. The lower real wage further reduces the output because rule-of-thumb agents consume their wages. So far, this reinforces the standard logic by which a positive shock to the real interest rate induces a fall in output. To invert this logic, one needs to decompose the consumption of savers into dividend and wage income. This can be seen directly from the budget constraint:

$$c_{s,t} = w_t + n_{s,t} + \frac{1}{1-\lambda} d_t, \tag{39}$$

where real profits d_t, accruing only to savers, are given by:

$$d_t = \frac{\mu}{1+\mu} y_t - w_t + a_t. \tag{40}$$

If the fall in the aggregate output y_t brought about by the interest rate increase is paired with a correspondingly larger decrease in wages, it leads to a positive income effect for savers (equation (40)). The positive income effect generates additional labor demand and the real wage eventually increases. In equilibrium, both the aggregate output and real wages increase. Hence, a higher real interest rate coincides with an increase in aggregate output induced by the positive income effect for savers. This mechanism implies that firm markups are countercyclical which is consistent with the empirical evidence (Rotemberg and Woodford, 1999).

VI.C. The link between consumption of savers and rule-of-thumb consumers

The negative δ implies that the consumption growth of rule-of-thumb consumers is negatively correlated with the variation in the real interest rate. Importantly, this link arises as a result of intertemporal choices of savers and does not imply that rule-of-thumb consumers have a negative EIS. Consistent with this prediction, using UK household-level data, Attanasio, Banks, and Tanner (2002) report a negative relationship for non-shareholders (Panel C of Table II in their paper). Additionally, Attanasio, Banks, and Tanner (2002) and Malloy, Moskowitz, and Vissing-Jorgensen (2009) also document that, both in the UK and the US, the time-series properties of stockholders' consumption growth differ substantially from the aggregate consumption growth. In particular, they show that the consumption of stockholders is more correlated with returns on risky assets than the aggregate consumption.

For the values of structural parameters that imply negative values for δ, the model implies that the consumption of savers loads negatively on the aggregate consumption and positively on technology shocks, which can be seen in equation (41) below:

$$c_{s,t} = \delta y_t + \zeta a_t, \tag{41}$$

where $\zeta > 0$ for $\lambda > 0$ and is increasing in λ:

$$\zeta = \frac{\lambda\eta(1+\kappa)}{(1-\lambda)\left[\kappa\lambda\eta - \lambda + \gamma\lambda(1+\kappa) + 1\right]},$$

(derivations of (41) and ζ are provided in Appendix E.2). Depending on the magnitude of δ and ζ, the consumption growth of savers is more volatile than the aggregate consumption growth. For $\delta < 0$ and moderate risk aversion parameter values, i.e. $\gamma < 10$, we have that $\zeta > 1$ as illustrated in Figure 9 for different levels of γ. With larger λ, a smaller fraction of savers holds all the risky assets in the economy and is thus more exposed to technology shocks.[25] Equation (41) does not imply that the consumption of savers is negatively correlated with the aggregate consumption in the data. The technology shock a_t is positively correlated with y_t and can induce a positive correlation in the data despite negative δ. Equation (41) also helps explain why the aggregate consumption appears too smooth when compared to substantial volatility of returns on risky assets. The model suggests that it is not driven by efficient risk sharing across agents but rather by intertemporal choices of savers which induce the negative correlation of their consumption with the total consumption.

VI.D. Why does the standard consumption Euler equation fail recover the EIS?

Irrespective of the preference specification, representative agent consumption-based asset pricing models fail to replicate the variation and the level of interest rates in the data (Weil, 1989). Following on this evidence, Canzoneri, Cumby, and Diba (2007) show that a broad range of preference specifications in consumption-based models imply a real short rate that is negatively correlated with the observed ex-post real interest rate. The model outlined in this paper offers one way to reconcile the mismatch between the consumption-based models and observed real rate dynamics. We do this by arguing that by estimating the aggregate Euler equation one does not recover the EIS but rather the EIS multiplied by a scaling constant δ that takes negative values. The log of nominal pricing kernel implied by the model reads:

$$m_{t,t+1} = \log(\beta) - \gamma \Delta c_{s,t+1} - \pi_{t+1}, \tag{42}$$

where $\Delta c_{s,t+1} = \log C_{s,t+1} - \log C_{s,t}$ denotes the consumption growth of savers who price all the assets. Using the differenced version of (41) and the market clearing condition $y_t = c_t$, rewrite (42) as:

$$m_{t,t+1} = \log(\beta) - \gamma\delta \Delta c_{t+1} - \gamma\zeta \Delta a_{t+1} - \pi_{t+1}. \tag{43}$$

[25]Setting $\lambda = 0$ implies $\delta = 1$ and $\zeta = 0$, which restores the representative agent setup.

The comparison of (42) with (43) provides the intuition for the mismatch between the observed short term interest rates and the pricing kernel. For $\delta < 0$, which is consistent with the empirical evidence, the aggregate consumption enters the pricing kernel with the opposite sign compared to the consumption of savers who are the only bond investors. The presence of rule-of-thumb consumers introduces a wedge between the aggregate Euler equation and the Euler equation of bond market participants (savers). Given that the consumption of savers is not directly observable, when equating the aggregate consumption growth with observed interest rates, one recovers $\frac{1}{\delta\gamma}$ rather than the EIS.

VII. Conclusions

We propose a new measure of the ex-ante real rate. In particular, we show that the slope of the term structure of interest rates is dominated by the variation in the ex-ante real rate. We argue that predictive regressions of real growth on the term structure slope are essentially an empirical implementation of the linearized aggregate output Euler equation. We further provide empirical evidence suggesting that time-varying risk premia and inflation expectations that the nominal slope could embed are unable to overturn our interpretation.

Building on these findings, we document that estimates of the EIS from the aggregate Euler equation are consistently negative and statistically significant, suggesting that the identification of the true EIS from aggregate data is a non-trivial task. The well-documented positive sign of the slope-growth relationship is consistent with our negative estimates of the EIS in aggregate data. The negative response of the aggregate consumption growth to an increase in the real rate might seem counterintuitive at first but it is sensible if one considers that a substantial fraction of aggregate consumption is attributed to agents whose financial wealth is negligible. These agents have little incentive to participate in asset markets to exploit changes in intertemporal trading opportunities and largely consume their wage income. This single friction can explain the negative relationship between the real rate and aggregate consumption (output) growth.

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A. Data

This section details the data series used to estimate the consumption and output Euler equations.

Yield data. We study US yields in the period January 1875 through December 2011 unless otherwise stated. The raw yield data are monthly and are obtained from the Global Financial Data (GFD) database. We consider the ten-year constant maturity yield (series IGUSA10D) as long term yield. To construct the short term yield for the whole sample period, we combine the three-month AA non-financial commercial paper yield (series IPUSAC3D) (1875-1933) with the three-month Treasury bill (1934-2009). The reason for using the three-month commercial paper in the earlier part of the sample is the data availability. The ten-year constant maturity yield is interpolated from available long term non-inflation-indexed Treasury securities. GFD itself combines various data sources such as the National Monetary Statistics from the Federal Reserve to obtain the ten-year CMT yield. $\tilde{r}_t$ is an average of monthly observations of $-\text{slope}_t$ within each year.

Consumption and output data (1875-2009). The consumption and output data are constructed by Robert Barro and Jose Ursua and downloaded from their website.[26] These data contain the total real per capita consumption/output index without further split into durable and non-durable consumption. Barro and Ursua compile the consumption data from various sources: For the period 1869-1899, the data are from Rhode (2002). For the 1900-1928 period, consumption data are taken from Lebergott (1996) and the period 1929-2009 is covered by data from the Bureau of Economic Analysis. The output data are sourced from Balke and Gordon (1989) for the period 1869-1928 and from BEA for the 1928-2009 period.

Output and non-Treasury interest rate data (1869-1983). The real GNP data for the period 1869-1983 are from Table I, Appendix B in Balke and Gordon (1986). The data in Balke and Gordon (1986) are assembled from various sources, see Notes on Section 1 of their paper for more details. The proxy for the real rate $\tilde{r}_t$ is computed as a difference between the commercial paper rate and yield on corporate bonds. The commercial paper has a maturity of six-months in most of the sample. Yields on corporate bonds are Baa-rated (Moodys) corporate bond yields (1919-1983) and railroad bond yields (1869-1918).

A.1. International data

Consumption and output data (1950-2009). Annual consumption and output data for Canada, Germany, Netherlands, Switzerland and the United Kingdom are obtained from the Barro and Ursua's data set.

Yield data. All yield data are from the GFD database, see their data descriptions for the original sources. For all countries, short-term nominal yield is represented by a three-month Treasury bill. The only exception is Switzerland in the period 1950-1979 where the commercial paper rate is used instead. The long-term yield is represented by a ten-year Treasury bond yield for Canada, Germany and Netherlands. For the UK, we use the 20-year government bond and the confederate long-term bond for Switzerland. $\tilde{r}_t$ is a difference between the three-month Treasury bill yield and the long-term bond yield averaged over monthly observations in a given year.

B. Robustness and additional results

[26] The dataset is available at `http://rbarro.com/data-sets/`.

B.1. Ex-ante real rate: robustness checks

We compare $\tilde{r}_t$ constructed using the nominal UK yield curve to the real interest rate obtained from the inflation-indexed government bonds. As shown in Figure 10, in the period 1985-2011, $\tilde{r}_t$ closely follows the real interest rate from UK inflation-indexed bonds with the correlation $\rho = 0.6$. The shortest continuously available maturity for the real interest rate is 3.5 years. Intuitively, the one-year real rate shall exhibit more variation than the one plotted in Figure 10 and would trace $\tilde{r}_t$ more closely.

B.2. Accuracy of the pre-1929 data

The interest rate and consumption data from the period before 1929 are not well-researched. Therefore, there is a potential issue that at least part of the results are driven by the selection of the dataset. To address these concerns, we evaluate the main result, namely the estimate of the EIS using alternative data sources.

First, we re-estimate the consumption Euler equation using the annual data by Robert Shiller.[27] The sample period is 1888-2009. $\tilde{r}_t$ is defined as the difference between the one-year interest rate and the long term government bond yield, which is a ten-year Treasury note post 1953. The results are quantitatively similar for the full sample and are statistically significant.

Second, between the Civil War and 1920, yields of government bonds are potentially downward-biased due to the fact that government bonds were held as reserves by banks and these could issue bank notes against them. To evaluate the potential bias, we use the non-Treasury yields to construct $\tilde{r}_t$ and use it to estimate the Euler equation. The results are quantitatively similar to those using Treasury yields.

C. EIS estimation: non-linear GMM and capital taxes

The main reason for estimating the log-linearized version of the Euler equation as opposed to estimating equation (14) via non-linear GMM (Hansen and Singleton, 1982) is that consumption is likely measured with an error. The measurement error can cause a relatively more severe bias in the non-linear estimation. This is especially relevant for the consumption data in the period before 1947. Romer (1986) shows that macro data before World War II are less accurate and are constructed using a different methodology than the post-World War II data.

We estimate the aggregate EIS from the pre-tax data. Mishkin (1981) argues that the effective tax rate varies substantially across households which makes it difficult to know the appropriate tax rate at macro level. Therefore, we do not adjust the real rate for capital income taxes which might bias the EIS estimates toward zero. Hence, our estimates of the EIS represent a lower bound on its magnitude.[28]

[27] http://www.econ.yale.edu/~shiller/data/chapt26.xls

[28] For illustration, the average marginal tax rate for capital income was slightly below 5% before World-War II, see Table 2 in Barro and Sahasakul (1983). Since the mid-1940s, the rate has been increasing to a level around 30%.

D. Empirical evidence on asset market participation

Wolff (2004) reports that the fraction of households with direct holdings of stocks increased from 13.10 percent in 1989 to 21.30 percent in 2001. Moreover, the share of stock-owning households including indirect holdings through mutual funds or pension accounts rose even more from 31.70 to 51.90 percent in the same period.

However, these numbers might misrepresent the degree of asset market participation for the purpose of our model. In the model, λ represents the fraction of households that actively participate in asset markets to smooth consumption which requires substantial liquid wealth. According to Wolff (2004), only 35 percent of households owned USD 10 000 (in 1995 dollars) or more of stocks in 2001. Using the Survey of Consumer Finances (SCF), Poterba and Samwick (1995) report that 24.60 and 29.30 percent of households had direct and indirect stock holdings of more than USD 2 000 (in 1992 dollars) in 1983 and 1992, respectively.

We use the SCF data to show that the financial wealth of low income households has not risen in real terms in the last two decades. Panel *a* of Figure 11 reports the fraction of stockholders by income quintiles for the period 1989-2007, and Panel *b* reports the corresponding median values of stock holdings adjusted for inflation. The important observation is that while the degree of participation has increased, the value of stock holdings for the lowest income quintile has remained virtually unchanged in the last two decades. In line with this evidence, Carroll (2000) reports that according to the SCF, the lowest 66 percentiles of the US population by wealth have liquid assets of only 10 percent of their annual wage income. Overall, these numbers indicate that $\lambda = 0.7$ is justified. The likely reason for the limited asset market participation are various forms of costs attached to it. Section 5 in Vissing-Jorgensen (2004) provides a thorough discussion of participation costs and their ability to explain the non-participation of a substantial portion of households.[29]

E. Model

This section provides details on some derivations of the model outlined in Section V.

E.1. Steady state

The steady state risk-less interest rate is $R = \frac{1}{\beta}$, the net mark-up is $\mu = \frac{1}{\varepsilon - 1}$ and define $F_Y = F/Y$. Fixed costs are introduced for convenience. Following Bilbiie (2008), we set the fixed costs share $F_Y = \mu$ which implies that profits are zero in steady-state. The share of the real wage on the total output is: $WN/PY = (1 + F_Y)/(1 + \mu)$, share of profit is $D/Y = (\mu - F)/(1 + \mu)$. Assume that both savers and rule-of-thumb consumers have the same hours worked in the steady state: $N_s = N_r = N$. It follows:

[29] Vissing-Jorgensen (2004) estimates a median per-period cost of participation of \$350 (real 1982-84 dollars).

$$\frac{C_s}{Y} = \frac{1+F_Y}{1+\mu} + \Theta\frac{\mu - F_Y}{1+\mu}$$

using the assumption $F_Y = \mu$, we have:

$$\frac{C_s}{Y} = 1$$

$$\frac{C_r}{Y} = \frac{1+F_Y}{1+\mu} = 1,$$

Hence, both types of agents have the same consumption in the steady state: $C_{s,t} = C_{r,t} = \left[\frac{N^\eta P}{W}\right]^{-1/\gamma}$.

E.2. Aggregate Euler equation

The aggregate Euler equation is derived by manipulating the the equilibrium conditions and budget constraints of both types of agents. First, we provide the derivation of the threshold level of market participation $\hat{\lambda}$ which is the discontinuity point at where the sign of the output Euler equation switches. Combining the budget constraint of rule-of-thumb consumers and their first order condition we get:

$$N_{r,t}^{\eta} = \left(\frac{W_t}{P_t} N_{r,t}\right)^{-\gamma} \frac{W_t}{P_t}$$

$$N_{r,t}^{\eta+\gamma} = \left(\frac{W_t}{P_t}\right)^{1-\gamma}.$$

Taking logs and rearranging yields:

$$n_{r,t} = \frac{1-\gamma}{\eta+\gamma} w_t. \tag{44}$$

To simplify the notation, define $\kappa \equiv \frac{1-\gamma}{\eta+\gamma}$. From the budget constraint of rule-of-thumb consumers we have: $c_{r,t} = (1+\kappa)w_t$. Total consumption c_t and hours worked n_t are defined as:

$$c_t = \lambda c_{r,t} + (1-\lambda)c_{s,t} \tag{45}$$

$$n_t = \lambda n_{r,t} + (1-\lambda)n_{s,t}. \tag{46}$$

Combining (44) and (46) with the labor supply of savers, we obtain the expression for the wage:

$$w_t = \frac{(1-\lambda)\gamma}{\eta\lambda\kappa + (1-\lambda)} c_{s,t} + \frac{\eta}{\eta\lambda\kappa + (1-\lambda)} n_t. \tag{47}$$

Plugging the consumption of rule-of-thumb consumers into (45) and using (47) yields:

$$c_t = \frac{\left[(1-\lambda)\left[\gamma\lambda\left(1+\kappa\right) + \left(\eta\lambda\kappa + (1-\lambda)\right)\right]\right]}{\eta\lambda\kappa + (1-\lambda)} c_{s,t} + \frac{\lambda\eta(1+\kappa)}{\eta\lambda\kappa + (1-\lambda)} n_t. \tag{48}$$

Expressing (48) in terms of output gives:

$$c_t = \frac{\left[(1-\lambda)\left[\gamma\lambda\left(1+\kappa\right) + \left(\eta\lambda\kappa + (1-\lambda)\right)\right]\right]}{\eta\lambda\kappa + (1-\lambda)} c_{s,t} + \frac{\lambda\eta(1+\kappa)}{\eta\lambda\kappa + (1-\lambda)} \frac{1}{1+\mu} y_t - \frac{\lambda\eta(1+\kappa)}{\eta\lambda\kappa + (1-\lambda)} a_t. \tag{49}$$

The differenced version of equation (49) is similar to the regression used in Campbell and Mankiw (1989) to estimate the fraction of rule-of-thumb households. The co-movement of consumption and output arises endogenously in the model. However, $\frac{\lambda\eta(1+\kappa)}{\eta\lambda\kappa+(1-\lambda)}\frac{1}{1+\mu}$ is not linear in λ, therefore linear regressions yield biased estimates of the fraction of rule-of-thumb households. The output Euler equation changes its sign when $\frac{\partial c_t}{\partial y_t} > 1$, therefore the threshold reads:

$$\begin{aligned} 1 &= \frac{\lambda\eta(1+\kappa)}{\eta\lambda\kappa + (1-\lambda)}\frac{1}{1+\mu} \\ \lambda\eta(1+\kappa) &= (1+\mu)\eta\lambda\kappa + (1-\lambda)(1+\mu) \\ \hat{\lambda} &= \frac{1+\mu}{1+\mu-(1+\mu)\eta\kappa+\eta(1+\kappa)} \\ &= \frac{1}{1+\frac{\eta(1-\kappa\mu)}{1+\mu}}. \end{aligned}$$

The risk aversion γ influences the threshold value $\hat{\lambda}$ in a non-linear way through κ. Rearrange (49) and impose the clearing condition $c_t = y_t$ to obtain:

$$c_{s,t} = \underbrace{\frac{(\eta\lambda\kappa + 1 - \lambda)(1+\mu) - \lambda\eta(1+\kappa)}{(1-\lambda)(1+\mu)[\kappa\lambda\eta + \gamma\lambda(1+\kappa)+1-\lambda]}}_{\delta} y_t + \underbrace{\frac{\lambda\eta(1+\kappa)}{(1-\lambda)[\kappa\lambda\eta - \lambda + \gamma\lambda(1+\kappa)+1]}}_{\zeta} a_t. \tag{50}$$

The Euler equation of savers reads:

$$\mathrm{E}_t c_{s,t+1} - c_{s,t} = \frac{1}{\gamma}(i_t - \mathrm{E}_t\pi_{t+1}), \tag{51}$$

substituting (50) into the Euler equation of savers obtains the aggregate Euler equation:

$$\mathrm{E}_t y_{t+1} - y_t = \frac{1}{\delta\gamma}(i_t - \mathrm{E}_t\pi_{t+1}) - \frac{1}{\frac{\kappa\lambda\eta+1-\lambda}{\lambda\eta(1+\kappa)} - \frac{1}{1+\mu}}[\mathrm{E}_t a_{t+1} - a_t] \tag{52}$$

$$\mathrm{E}_t y_{t+1} - y_t = \frac{1}{\delta\gamma}(i_t - \mathrm{E}_t\pi_{t+1}) + \chi[\mathrm{E}_t a_{t+1} - a_t], \tag{53}$$

where $\chi \equiv -\frac{1}{\frac{\kappa\lambda\eta+1-\lambda}{\lambda\eta(1+\kappa)} - \frac{1}{1+\mu}}$.

E.3. Natural real rate

We start by deriving the relationship between the real wage and total output. Combining (50) with the production function given by $y_t = (1+\mu)a_t + (1+\mu)n_t$ and the labor supply of savers given by $\eta n_{s,t} = w_t - \gamma c_{s,t}$ and rearranging obtains:

$$w_t = \underbrace{\frac{\eta + (1+\mu)(1-\lambda)\gamma\delta}{(1+\mu)(1-\lambda+\lambda\eta\kappa)}}_{\vartheta} y_t + \underbrace{\frac{(1-\lambda)\gamma\zeta - \eta}{1-\lambda+\lambda\eta\kappa}}_{\nu} a_t. \tag{54}$$

The New Keynesian Phillips curve reads:

$$\pi_t = \beta\mathrm{E}_t\pi_{t+1} + \psi mc_t, \tag{55}$$

with $\psi \equiv \frac{(1-\omega\beta)(1-\omega)}{\omega}$. Real marginal costs can be expressed as $mc_t = w_t - a_t$. Using (54), mc_t can be expressed as:

$$mc_t = \vartheta y_t + (\nu - 1)a_t, \tag{56}$$

therefore:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \psi \left[\vartheta y_t + (\nu - 1)a_t\right]. \tag{57}$$

The natural level of output y_t^* is obtained from (57) by setting the inflation to zero:

$$y_t^* = \frac{1}{\vartheta}(1-\nu)a_t, \tag{58}$$

the Phillips curve can be rewritten in terms of output gap $x_t \equiv y_t - y_t^*$:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \psi\vartheta x_t. \tag{59}$$

From the IS curve given by (53) evaluated at zero inflation one obtains the natural level of the real interest rate:

$$r_t^* = \delta\gamma \left(\frac{1}{\vartheta}(1-\nu) - \chi\right) \left[\mathrm{E}_t a_{t+1} - a_t\right]. \tag{60}$$

The IS curve in terms of output gap reads:

$$\mathrm{E}_t x_{t+1} - x_t = \frac{1}{\delta\gamma}\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^*\right) \tag{61}$$

E.4. Model solution

We collect variables into a vector: $z_t = [\pi_t, y_t, i_t, y_t^*]'$. The equilibrium conditions are stated in matrix form:

$$\begin{bmatrix} 1 & -\psi\vartheta & 0 & \psi\vartheta \\ 0 & 1 & \frac{1}{\delta\gamma} & 0 \\ -(1-\phi_i)\phi_\pi & -(1-\phi_i)\phi_x & 1 & (1-\phi_i)\phi_x \\ 0 & 0 & 0 & 1 \end{bmatrix} z_t = \begin{bmatrix} \beta & 0 & 0 & 0 \\ \frac{1}{\delta\gamma} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathrm{E}_t z_{t+1} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_i & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} z_{t-1} + \tag{62}$$

$$+ \begin{bmatrix} 0 \\ -\chi(\rho_a - 1) \\ 0 \\ \frac{1}{\vartheta}(1-\nu) \end{bmatrix} a_t. \tag{63}$$

In a compact form:

$$B_1 z_t = A \mathrm{E}_t z_{t+1} + B_2 z_{t-1} + C a_t, \tag{64}$$

where matrices A, B_1, B_2, C are implicitly defined by comparing (63) with (64). The rational expectations equilibrium follows:

$$z_t = \Gamma z_{t-1} + \Xi a_t. \tag{65}$$

The model is solved by the forward method proposed in Cho and Moreno (2011). For convenience, we define $X_t = (z_t', a_t)'$ and rewrite (65) in a compact form:

$$X_t = \varpi X_{t-1} + \Psi \varepsilon_t^a, \tag{66}$$

where $\varpi = \begin{bmatrix} \Gamma & \Xi\rho_a \\ \mathbf{0}_{1\times 4} & \rho_a \end{bmatrix}$ and $\Psi = \begin{bmatrix} \Xi \\ 1 \end{bmatrix}$.

E.5. Affine term structure model

In setting up the affine term structure model, we follow Bekaert, Cho, and Moreno (2010) and keep the risk premia constant. Given that the paper focuses on intertemporal trade-off and expectations, time-varying risk premia are of secondary importance. The log of the nominal pricing kernel implied by the model outlined in Section V, reads:

$$m_{t,t+1} = \log(\beta) - \gamma \Delta c_{s,t+1} - \pi_{t+1}, \tag{67}$$

where $\Delta c_{s,t+1} = \log C_{s,t+1} - \log C_{s,t}$ denotes the consumption growth of savers. Assuming log-normality, the pricing equation for short-term bonds implies:

$$\mathrm{E}_t m_{t,t+1} + \frac{1}{2}\mathrm{Var}_t m_{t,t+1} = -i_t. \tag{68}$$

Rewrite (67) using the relationship between the consumption of savers and the total output given in (50):

$$m_{t,t+1} = \log(\beta) - \gamma\delta\Delta y_{t+1} - \gamma\zeta\Delta a_{t+1} - \pi_{t+1}. \tag{69}$$

Define:

$$\Lambda' = \begin{bmatrix} 1 & \gamma\delta & 0 & 0 & \gamma\zeta \end{bmatrix} \Psi,$$

and Var $\varepsilon_t^a = D$, then $\mathrm{Var}_t m_{t,t+1} = \Lambda' D \Lambda$. Given the assumptions, $m_{t,t+1}$ can be written as:

$$m_{t,t+1} = -i_t - \frac{1}{2}\Lambda' D\Lambda + \Lambda' \epsilon_{t+1}, \tag{70}$$

and the n-period zero coupon yield reads:

$$\begin{aligned} y_t^{(n)} &= -\frac{A_n}{n} - \frac{B_n}{n} X_t \\ A_n &= A_{n-1} + \frac{1}{2} B_{n-1}' \Psi D \Psi' B_{n-1} - \Lambda' D \Psi' B_{n-1} \\ B_n &= B_{n-1}' \varpi \\ B_1 &= -e_3'. \end{aligned}$$

To test if the model-implied term structure recovers the short term real rate, we simulate the model at the parameters given in Table VII for 220 quarters and construct the yield curve. Then, we construct the measure of the real rate $\tilde{r}_t = i_t - y_t^{(10)}$ as in the previous sections and compare it to $i_t - \mathrm{E}_t \pi_{t+1}$ where $\mathrm{E}_t \pi_{t+1}$ is obtained from an AR(1) model.

Table I: Forecasting output growth, 1972-2013

Panel A reports the regression results for forecasting the output growth denoted by $g_{t+k/4}$ with the slope where the horizon k varies between one and eight quarters. Panel B reports the results using orthogonalized slope (as estimated in equation (5)) and a measure of risk premia $(\widehat{cf}_t)$ as predictors. Panel C differs from B by using the Cochrane-Piazzesi factor CP_t as a measure of risk premia. The sample period is Q1:1972–Q4:2013. Both the sampling frequency and beginning of the sample period are determined by the availability of the bond risk premium proxy. Newey-West adjusted t-statistics are reported in parentheses, the number of lags is 12.

	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
A. $\Delta g_{t+k/4} = \alpha_0 + \alpha_1 \textbf{slope}_t + \varepsilon_{t+k/4}$								
α_1	0.19	0.38	0.55	0.78	0.94	1.10	1.23	1.33
	(3.76)	(3.91)	(4.02)	(4.45)	(4.54)	(4.78)	(4.98)	(5.04)
$\bar{R}^2$	0.09	0.13	0.15	0.20	0.22	0.24	0.25	0.25
B. $\Delta g_{t+k/4} = \alpha_0 + \alpha_1 \textbf{slope}_t^{\perp,cf} + \alpha_2 \widehat{cf}_t + \varepsilon_{t+k/4}$								
α_1	0.28	0.57	0.82	1.12	1.33	1.52	1.65	1.72
	(3.15)	(3.33)	(3.54)	(3.82)	(4.04)	(4.26)	(4.36)	(4.27)
α_2	0.03	0.03	0.06	0.21	0.35	0.56	0.80	1.08
	(0.32)	(0.17)	(0.24)	(0.66)	(0.94)	(1.27)	(1.57)	(1.91)
$\bar{R}^2$	0.12	0.18	0.20	0.26	0.28	0.30	0.30	0.29
C. $\Delta g_{t+k/4} = \alpha_0 + \alpha_1 \textbf{slope}_t^{\perp,cp} + \alpha_2 CP_t + \varepsilon_{t+k/4}$								
α_1	0.31	0.58	0.82	1.14	1.28	1.42	1.45	1.50
	(3.76)	(3.61)	(3.75)	(4.02)	(3.92)	(3.94)	(3.67)	(3.60)
α_2	0.01	0.03	0.05	0.08	0.12	0.15	0.20	0.23
	(0.99)	(1.25)	(1.39)	(1.67)	(2.14)	(2.56)	(2.92)	(3.12)
$\bar{R}^2$	0.11	0.16	0.18	0.24	0.25	0.27	0.27	0.27

Table II: Bond excess returns and the nominal term spread

Panel A reports basic statistical properties of the nominal term spread across sub-samples. Panel B reports the regression results for a univariate predictive regression of excess returns on a ten-year treasury bond at an annual horizon $rx_{t+1}^{(10)}$ on the nominal term spread defined as ten-year nominal Treasury yield less three-month yield. Panel C reports the regression results with the nominal term and the dummy variable D_t indicating the reserve-targeting period (1979-1984), Panel D reports the results for the alternative dating of the reserve-targeting period (1979-1982). The sample period is 1875-2011 and the data are annual. Sub-samples are selected such that they correspond to samples used in the literature. Standard errors in Panel B are obtained with Newey-West adjustment with four lags and both sides of the regression are standardized.

	1875-2011	1875-1979	1929-2011	1952-2011	1971-2011	1982-2011
Panel A. Statistical properties of the nominal term spread						
Mean (%)	0.23	-0.24	1.36	1.32	1.60	1.90
St. dev. (%)	1.91	1.81	1.17	1.20	1.33	1.20
AR(1Y)	0.67	0.62	0.53	0.52	0.49	0.45
Number of obs.	136	105	82	59	40	29
Panel B. $rx_{t+1}^{(10)} = \alpha_0 + \alpha_1 \mathbf{slope}_t + \varepsilon_{t+1}$						
α_1	0.22	0.10	0.33	0.35	0.34	0.12
	(2.60)	(0.87)	(3.14)	(3.33)	(2.63)	(0.98)
$\bar{R}^2$	0.04	0.00	0.10	0.11	0.09	-0.02
Panel C. $rx_{t+1}^{(10)} = \alpha_0 + \alpha_1 \mathbf{slope}_t + \alpha_2 D_t^{reserv} \times \mathbf{slope}_t + \varepsilon_{t+1}$						
α_1	0.16	–	0.23	0.24	0.20	–
	(2.08)	–	(2.62)	(2.49)	(1.70)	–
α_2	1.08	–	0.54	0.45	0.47	–
	(2.84)	–	(4.05)	(3.36)	(3.29)	–
$\bar{R}^2$	0.10	–	0.13	0.13	0.11	–
Panel D. $rx_{t+1}^{(10)} = \alpha_0 + \alpha_1 \mathbf{slope}_t + \alpha_2 D_t^{reserv} \times \mathbf{slope}_t + \varepsilon_{t+1}$						
α_1	0.19	–	0.26	0.28	0.24	–
	(2.33)	–	(2.81)	(2.76)	(1.97)	–
α_2	1.08	–	0.44	0.35	0.37	–
	(1.56)	–	(2.53)	(2.00)	(2.25)	–
$\bar{R}^2$	0.07	–	0.11	0.11	0.09	–

Table III: Nominal term spread and inflation expectations

This table reports the regression results for a univariate predictive regression of changes in inflation $\pi_{t+h+1} - \pi_{t+1}$ on the nominal term spread defined as ten-year nominal Treasury yield less three-month yield. The sample period is 1875-2011 and the data are annual. Standard errors are obtained with Newey-West adjustment with four lags. Both sides of the regression are standardized. Panels A through E report the results for horizons one to five years.

$\pi_{t+h+1} - \pi_{t+1} = \alpha_0 + \alpha_1 \text{slope}_t + \varepsilon_{t+h+1}$						
	1875-2011	1875-1979	1929-2011	1952-2011	1971-2011	1982-2011
Panel A. h=1						
α_1	0.06	0.06	0.15	0.38	0.46	0.16
	(0.95)	(0.91)	(1.46)	(2.33)	(2.85)	(1.16)
$\bar{R}^2$	0.00	-0.01	0.01	0.13	0.19	-0.01
Panel B. h=2						
α_1	0.07	0.07	0.17	0.43	0.57	0.24
	(0.95)	(0.92)	(1.34)	(2.25)	(2.89)	(1.46)
$\bar{R}^2$	0.00	-0.01	0.02	0.17	0.31	0.02
Panel C. h=3						
α_1	0.06	0.05	0.17	0.42	0.57	0.24
	(0.76)	(0.64)	(1.18)	(2.06)	(2.72)	(1.51)
$\bar{R}^2$	0.00	-0.01	0.02	0.16	0.30	0.02
Panel D. h=4						
α_1	0.03	0.02	0.15	0.38	0.56	0.26
	(0.44)	(0.22)	(1.00)	(1.84)	(2.62)	(1.31)
$\bar{R}^2$	-0.01	-0.01	0.01	0.13	0.29	0.02
Panel E. h=5						
α_1	0.02	-0.00	0.14	0.34	0.54	0.15
	(0.18)	(-0.00)	(0.80)	(1.52)	(2.26)	(0.70)
$\bar{R}^2$	-0.01	-0.01	0.01	0.10	0.27	-0.02

Table IV: Estimation of consumption and output Euler equation, 1875–2009

The table reports estimation results for the log-linearized Euler equation given by equation (14). The setup is implemented as two-stage least squares. Each panel reports the results for two different instrument sets used for the ex-post real rate. *Instrument set 1* contains only the slope of the term structure, *Instrument set 2* consists of the nominal term spread, its lag, nominal three-month T-bill, past consumption growth, and past inflation. Panel A reports the results for the total consumption growth and Panel B for the output growth. The data are annual. The robust standard errors are reported in parentheses. The over-identification test refers to Hansen's J statistic. The "Weak identification test" in each panel reports the F-statistic (Kleibergen-Paap). The critical values for the maximal size are as follows. Instrument set 1: 16.38 for 10%, 8.96 for 15%, 6.66 for 20%; Instrument set 2: 26.87 for 10%, 15.09 for 15%, 10.98 for 20%. The critical values for relative bias are as follows. Instrument set 2: 18.37 for 5%, 10.83 for 10%, 6.77 for 20%. The critical values are computed using the procedure proposed by Stock and Yogo (2002).

	Instrument set 1	Instrument set 2
A. Consumption Euler equation		
$\widehat{\sigma}$	-0.44	-0.06
Robust s.e.	(0.15)	(0.10)
Confidence interval (95%)	[-0.74, -0.14]	[-0.25, 0.13]
Overidentification test (p-val)	–	0.01
Weak identification test (F-stat)	36.41	22.37
B. Output Euler equation		
$\widehat{\sigma}$	-0.56	-0.27
Robust s.e.	(0.20)	(0.15)
Confidence interval (95%)	[-0.95, -0.17]	[-0.57, 0.02]
Overidentification test (p-val)	–	0.26
Weak identification test (F-stat)	36.41	22.37

Table V: Estimation of consumption Euler equation, 1960–2013

The table reports estimation results for the log-linearized Euler equation given by (14). Each panel reports the results for two different instrument sets used for the ex-post real rate. *Instrument set 1* contains the slope of the term structure, *Instrument set 2* consists of the slope, its lag, nominal three-month T-bill, past consumption growth, and past inflation. Panel A reports the results for non-durable consumption and services, Panel B for the durable consumption expenditures, Panel C for the total consumption, and Panel D for the output. The data are quarterly. The robust standard errors are reported in parentheses. The over-identification test refers to Hansen's J statistic. The "Weak identification test" in each panel reports the F-statistic (Kleibergen-Paap). The critical values for the maximal size are as follows. Instrument set 1: 16.38 for 10%, 8.96 for 15%, 6.66 for 20%; Instrument set 3: 26.87 for 10%, 15.09 for 15%, 10.98 for 20%. The critical values for relative bias are as follows. Instrument set 2: 18.37 for 5%, 10.83 for 10%, 6.77 for 20%. The critical values are computed using the procedure proposed by Stock and Yogo (2002).

	Instrument set 1	Instrument set 2
A. Nondurable consumption & services		
$\widehat{\sigma}$	-0.42	0.08
Robust s.e.	(0.24)	(0.07)
Confidence interval (95%)	[-0.90, 0.05]	[-0.06, 0.23]
Overidentification test (p-val)	–	0.00
Weak identification test (F-stat)	19.61	212.68
B. Durable expenditures		
$\widehat{\sigma}$	-3.97	-0.25
Robust s.e.	(1.58)	(0.52)
Confidence interval (95%)	[-7.06, -0.88]	[-1.27, 0.76]
Overidentification test (p-val)	–	0.03
Weak identification test (F-stat)	19.61	213.58
C. Total consumption		
$\widehat{\sigma}$	-0.68	0.05
Robust s.e.	(0.31)	(0.09)
Confidence interval (95%)	[-1.29, -0.08]	[-0.13, 0.23]
Overidentification test (p-val)	–	0.00
Weak identification test (F-stat)	19.61	213.732
D. Output		
$\widehat{\sigma}$	-0.82	-0.04
Robust s.e.	(0.40)	(0.14)
Confidence interval (95%)	[-1.61, -0.03]	[-0.32, 0.23]
Overidentification test (p-val)	–	0.00
Weak identification test (F-stat)	19.75	216.77

Table VI: Euler equation estimates for selected developed countries

The table reports estimation results for the log-linearized Euler equation given by (14) estimated using $\tilde{r}_t$ as a single instrument for the ex-post real rate. Panel A reports the results for consumption and Panel B for real output. Each column corresponds to a different country. The sample period for each country is 1950–2009, the only exception is Germany with the sample period 1956–2009 (due to the data availability). The data are annual and their sources are described in Appendix A.1. The robust standard errors are reported in parentheses. The "Weak identification test" in each panel reports the F-statistic (Kleibergen-Paap). The critical values for the maximal size are 16.38 for 10%, 6.66 for 20%, and 5.53 for 25%. The critical values are computed using the procedure proposed by Stock and Yogo (2002).

	Canada	Germany	Netherlands	Switzerland	United Kingdom
A. Consumption Euler equation					
$\widehat{\sigma}$	-0.74	-1.44	-0.76	-0.57	-0.12
Robust s.e.	(0.21)	(0.53)	(0.21)	(0.27)	(0.14)
Confidence interval (95%)	[-1.14, -0.33]	[-2.48, -0.40]	[-1.16, -0.35]	[-1.10, -0.04]	[-0.40, 0.16]
Weak identification test (F-stat)	19.52	8.22	18.94	4.77	41.28
B. Output Euler equation					
$\widehat{\sigma}$	-0.84	-1.43	-0.50	-1.21	-0.26
Robust s.e.	(0.24)	(0.63)	(0.15)	(0.60)	(0.09)
Confidence interval (95%)	[-1.31, -0.36]	[-2.67, -0.19]	[-0.79, -0.20]	[-2.38, -0.05]	[-0.45, -0.07]
Weak identification test (F-stat)	19.52	8.22	18.94	4.77	41.28

Table VII: Model parameters

This table collects the parameter values used to calibrate the model. All parameters refer to quarterly frequency.

Parameter		Value	Reference
Preference parameters			
Discount factor	β	0.99	–
Risk aversion	γ	2	Chetty (2006)
Labor elasticity parameter	η	1.33	Chetty, Guren, Manoli, and Weber (2011, 2012)
Price rigidities			
Steady state markup	μ	0.2	Eichenbaum and Fisher (2007)
Calvo parameter	ω	0.75	Eichenbaum and Fisher (2007)
Limited participation			
Degree of limited participation	λ	0.7	Vissing-Jorgensen (2002); Guvenen (2006)
Interest rate rule			
Interest rate smoothing	ϕ_i	0.85	Coibion and Gorodnichenko (2012)
Inflation sensitivity	ϕ_π	0.4	
Output gap sensitivity	ϕ_x	1.2	
Exogenous shocks			
Persistence of technology shock	ρ_a	0.3	Fernald (2012)
Volatility of technology shock	σ_a	0.01	Fernald (2012)

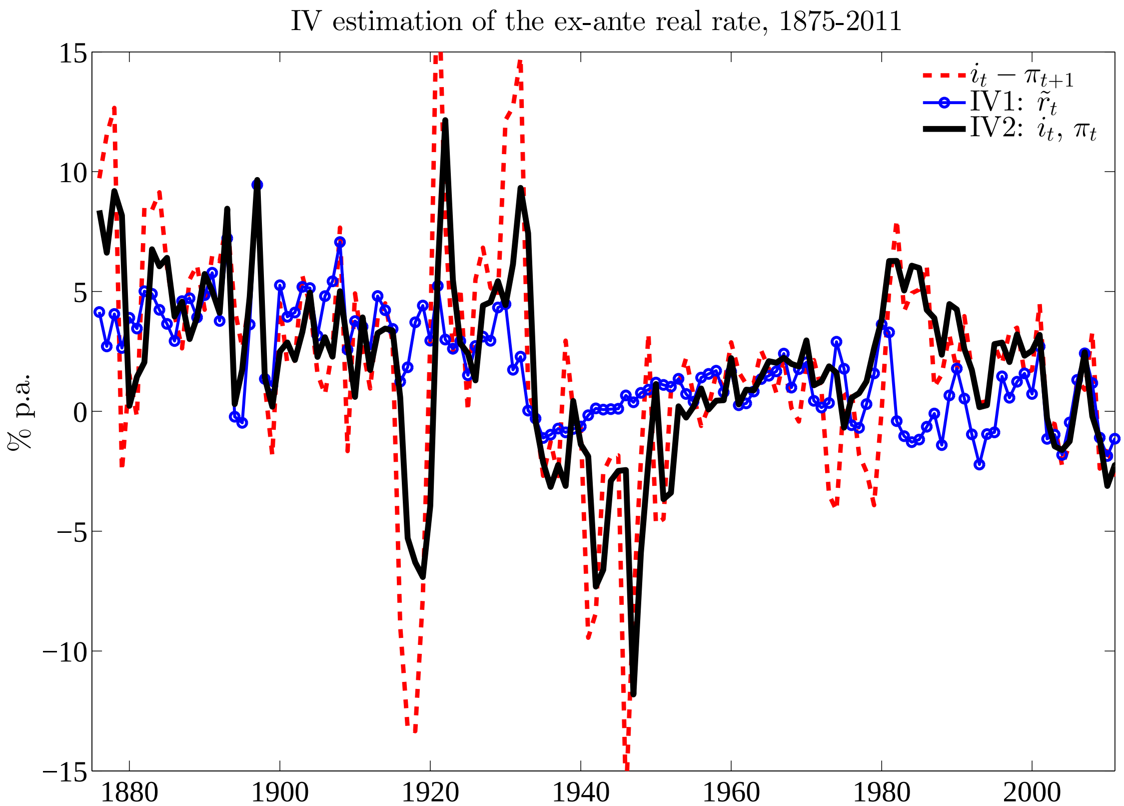
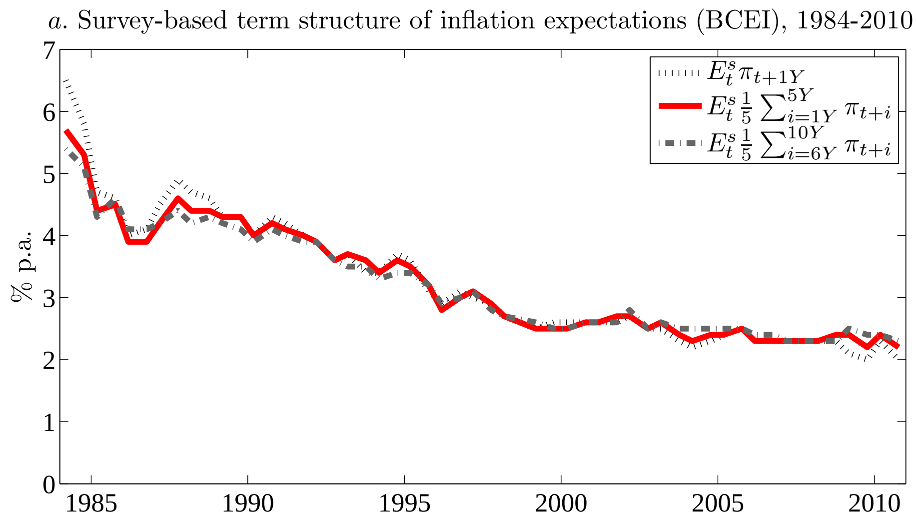


Figure 1: Instrumental variables estimation of the ex-ante real interest rate, 1875-2011

The figure compares two estimates of the ex-ante real rate, each obtained with a different set of instruments. IV1: the instrument set includes only $\tilde{r}_t$ (circled blue line). IV2: the ex-post real rate $i_t - \pi_{t+1}$ is regressed on the nominal short-term interest rate i_t represented by the three-month nominal T-bill and past inflation π_t (black line). The dashed red line represents the ex-post real rate. The data are annual and the sample period is 1875-2011.



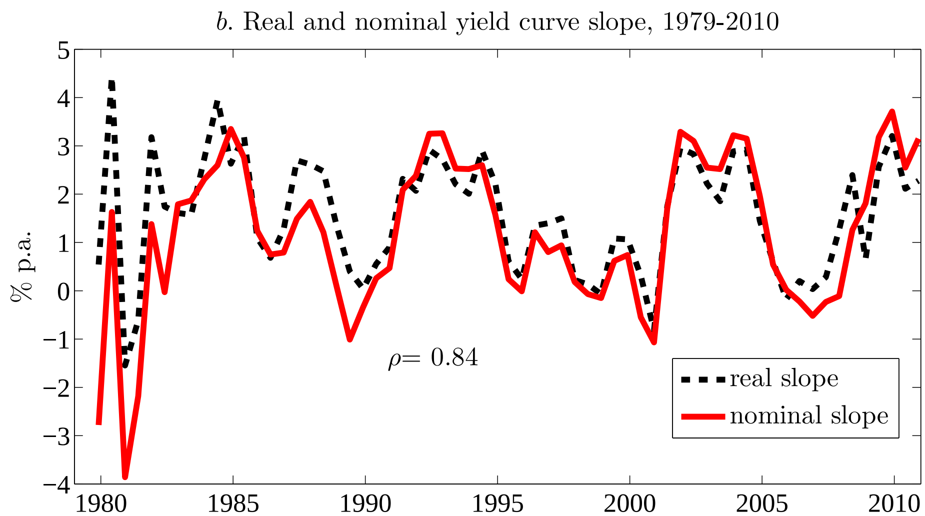


Figure 2: Term structure of inflation expectations

Panel a plots the survey-based inflation expectations obtained from the Blue Chip Economic Indicators survey. The data are semi-annual as the survey is conducted in March and October each year. The sample period is 1984 through 2010. $E_t^s \pi_{t+1Y}$ denotes the median survey response about the inflation one year ahead; $E_t^s \frac{1}{5} \sum_{i=1Y}^{5Y} \pi_{t+i}$ is the median survey response about the average inflation over the next five years, $E_t^s \frac{1}{5} \sum_{i=6Y}^{10Y} \pi_{t+i}$ is the median response about the average inflation between five and ten years ahead. Panel b compares the real and nominal slope of the yield curve. The real slope is constructed as the difference of the ten-year real yield and the real three-month Treasury bill. The real ten-year yield is obtained using ten-year inflation expectations from Livingston and SPF surveys obtained from the Philadelphia Fed. The data are semi-annual. The sample period is 1979-2010. The start of the sample is dictated by the availability of the long term inflation survey data.

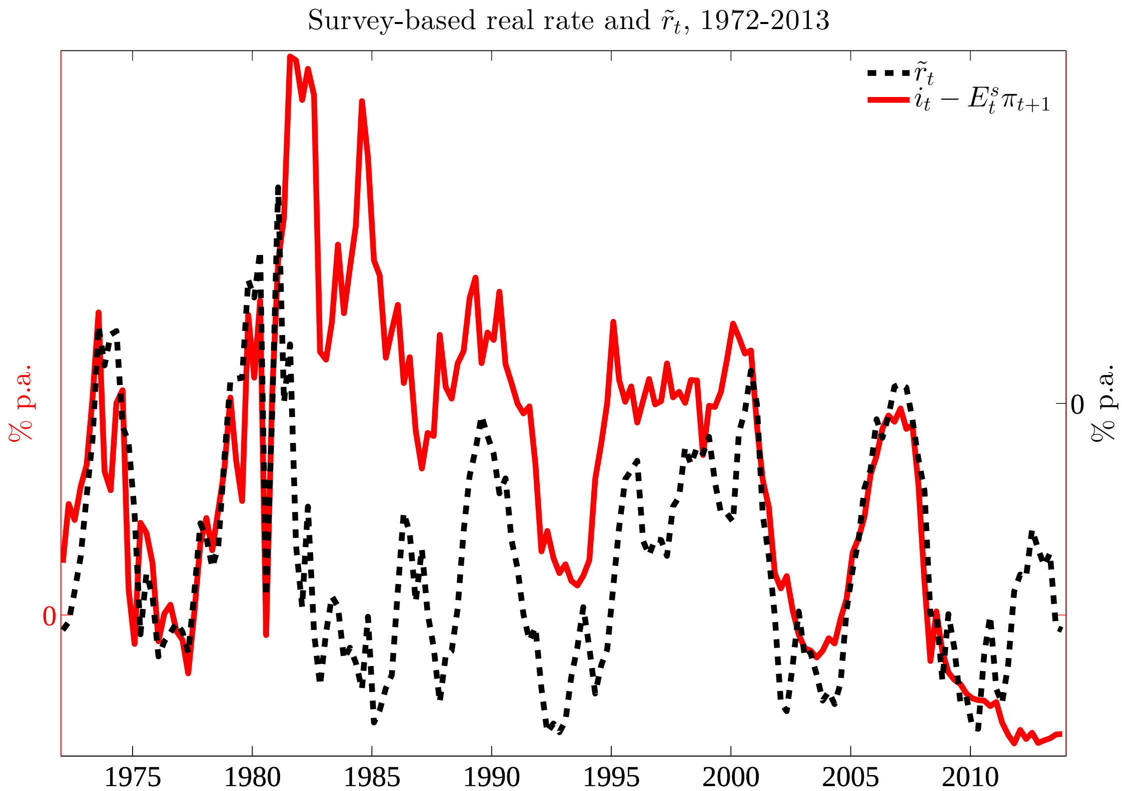


Figure 3: Survey-based ex-ante real interest rate and $\tilde{r}_t$, 1972-2013

The figure plots the measure of the ex-ante real rate $\tilde{r}_t$, which is computed as the difference between the three-month Treasury bill yield and ten-year Treasury yield together with the survey-based real interest rate $r_t^{surv} = i_t - \mathrm{E}_t^s \pi_{t+1}$, where i_t represents the three-month Treasury bill and $\mathrm{E}_t^s \pi_{t+1}$ denotes the survey-based measure of inflation expectations over the next 12 months. The survey data are obtained from the SPF survey with the median response being a proxy for expectations. The data are quarterly. The sample period is Q1:1972–Q4:2013.

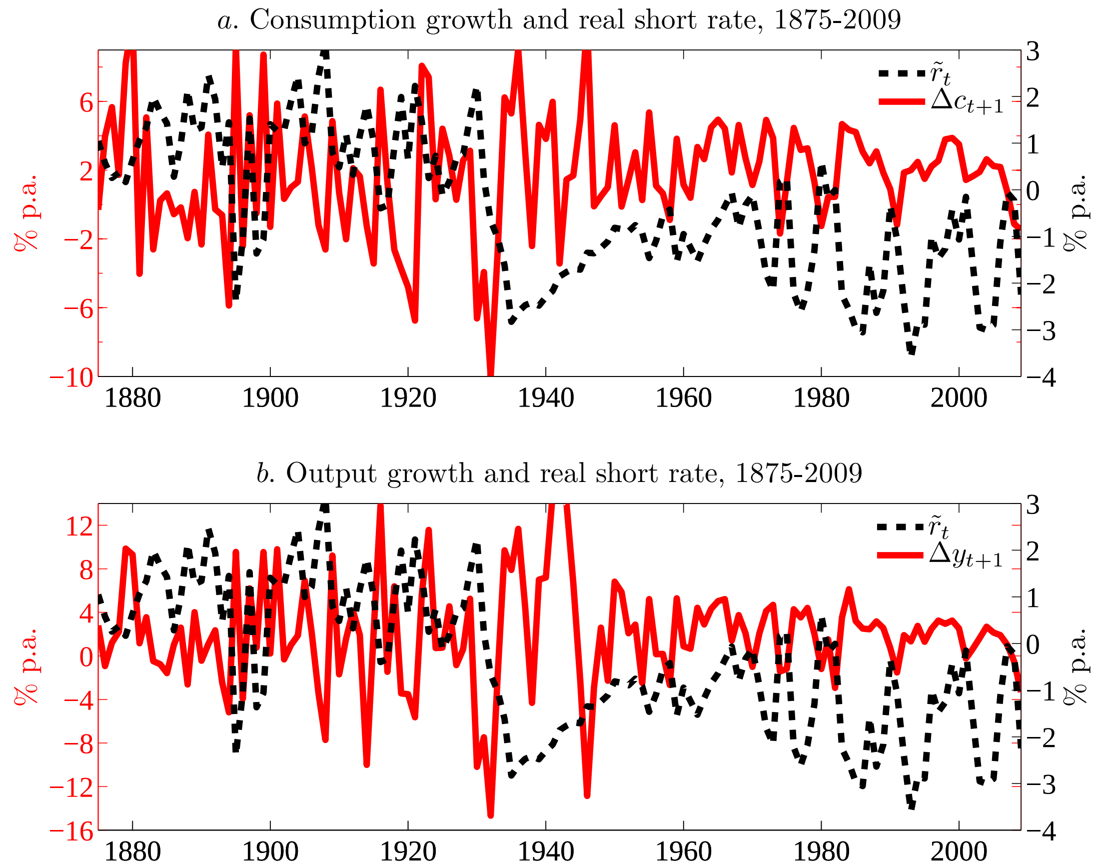


Figure 4: Consumption and output growth and the real interest rate, 1875–2009

Panel a plots real per capita consumption growth superimposed with the proxy for the short term real interest rate denoted by $\tilde{r}_t$. Panel b plots the real output growth superimposed with $\tilde{r}_t$. The data are annual and the sample period is 1875 through 2009.

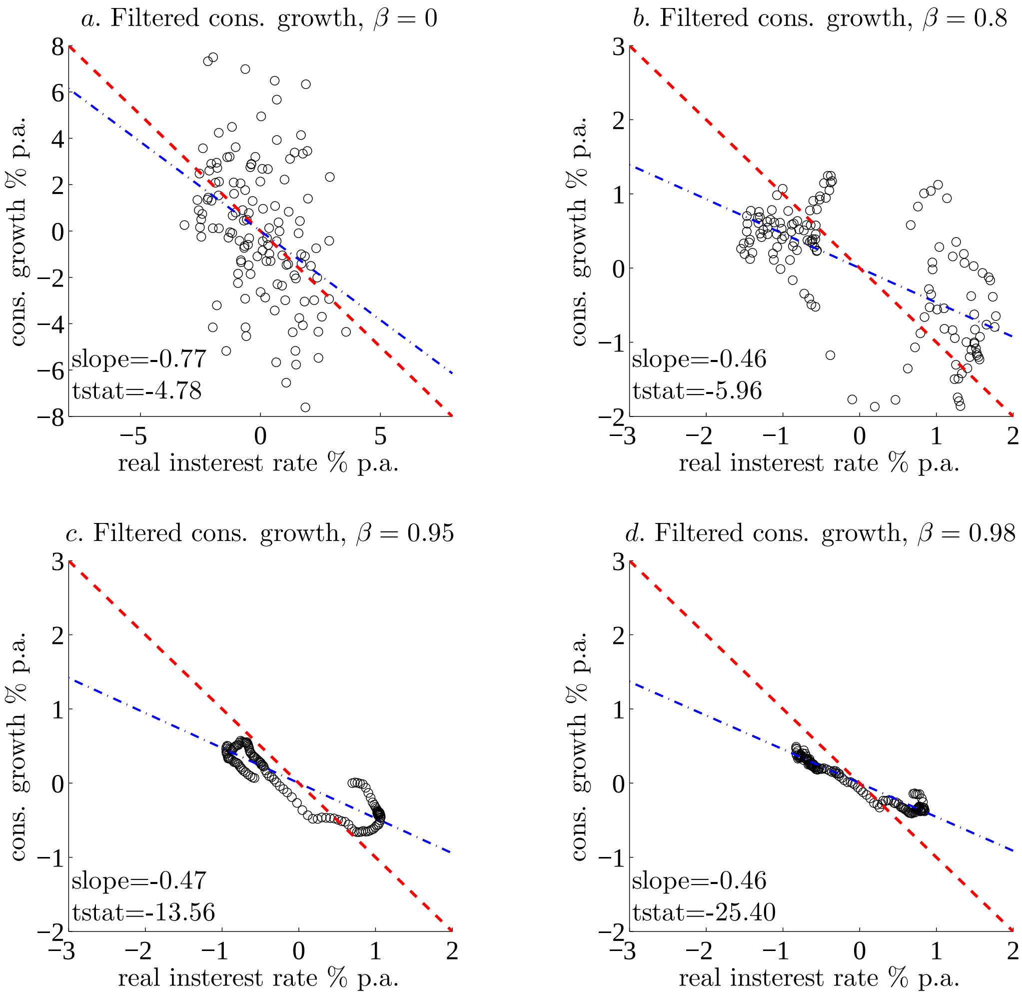


Figure 5: Consumption growth and real interest rate–Lucas filter, 1875–2009

Panels a–d scatter-plot the filtered consumption growth and the real rate measure $\tilde{r}_t$ for different filtering parameters β. For $\beta = 0$ (Panel a), the filter corresponds to a linear regression of unfiltered series. The filter is given by equation (16). We set $n = 50$. Blue dash-dotted lines represent the regression coefficient obtained from a regression of the filtered consumption growth on filtered $\tilde{r}_t$. The regression coefficients together with Newey-West adjusted t-statistics are reported in the lower left corner of each panel. The red dashed line represents the regression slope of minus one. The data are annual and the sample period is 1875 through 2009.

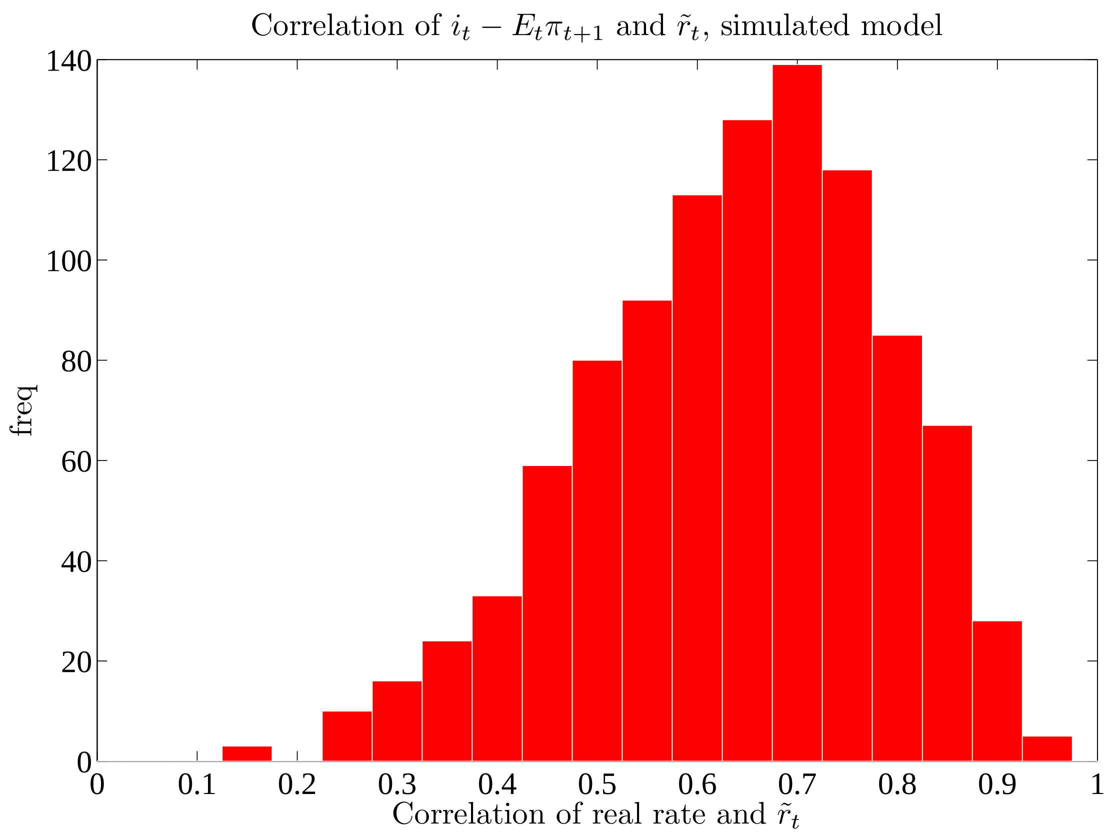


Figure 6: Correlation of $\tilde{r}_t$ and real rate in a simulated model.

The figure reports the distribution of the correlation between $\tilde{r}_t$ and the ex-ante real interest rate from a simulated model. $\tilde{r}_t$ is obtained as a difference between the nominal short rate and the ten-year yield. Ex-ante real rate is computed as a difference between short term nominal yield and expected inflation. To obtain the expected inflation, we estimate an AR(1) model on realized inflation and forecast two quarters ahead. The model is simulated for 220 quarters with 1000 replications.

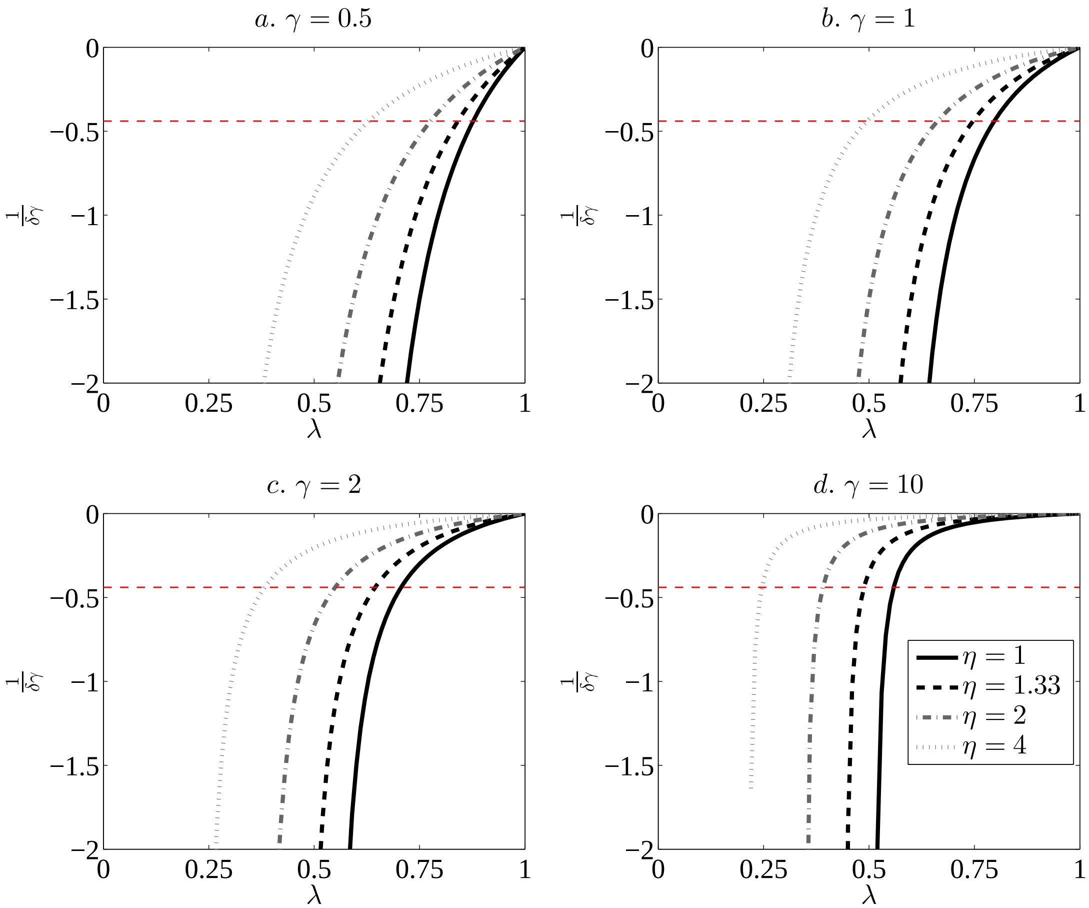


Figure 7: Parameterizations of the aggregate output Euler equation.

Panels $a - d$ plot the parameter values for $\frac{1}{\delta\gamma}$ in the aggregate output Euler equation given by (33). Three structural parameters are varied. First, the coefficient of relative risk aversion γ takes values between 0.5 and 10. Each panel represents different values of γ. Second, the Frisch elasticity of labor supply is varied between 0.25 and 1 which implies parameter values for η between 1 and 4. Third, the share of rule-of-thumb consumers λ varies between 0 and 1 (x-axis). The red dashed line represents the estimate of $\frac{1}{\delta\gamma}$ based on the annual data for total consumption and interest rates in the period 1875-2009, see the first column of Panel A in Table IV.

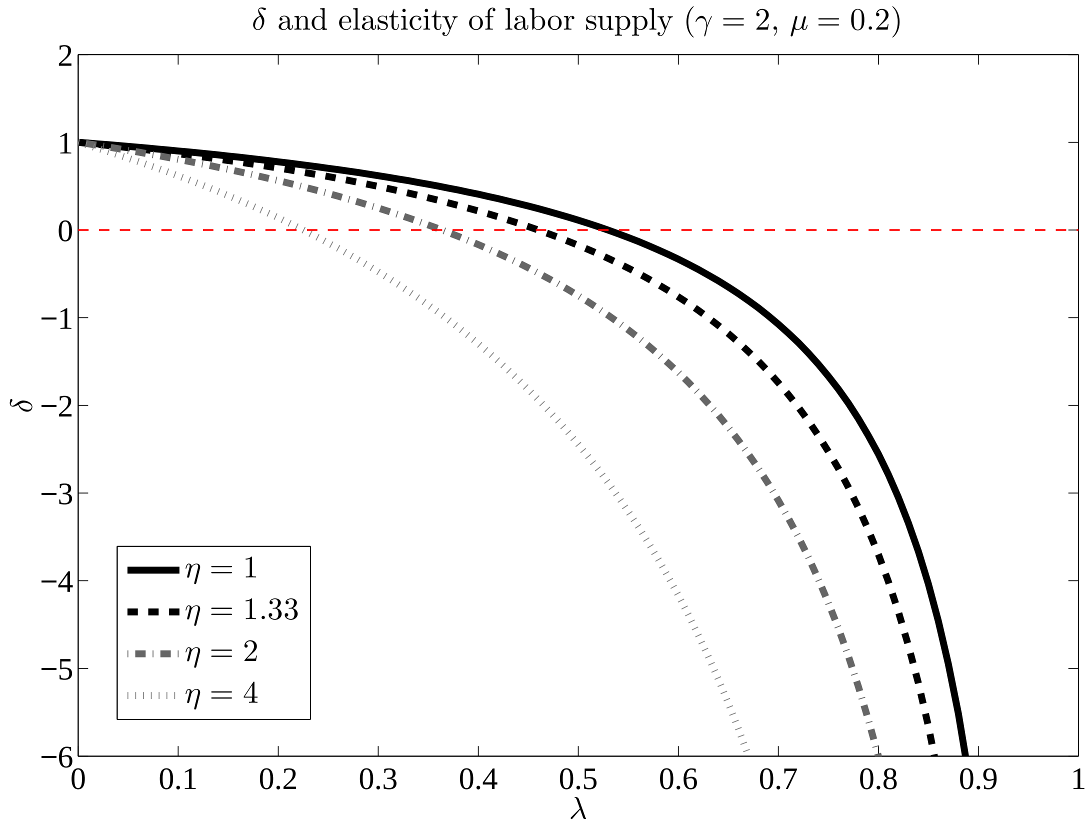


Figure 8: Parameterization of δ.

The figure shows the dependence of δ on the elasticity of labor supply parameter η and the asset market participation parameter λ. Expression for δ is given by (34). Note that η is the inverse of the Frisch elasticity of labor supply. Hence, the elasticity of labor supply is varied between 0.25 and 1. The risk aversion parameter is $\gamma = 2$ and the steady-state markup is $\mu = 0.2$.

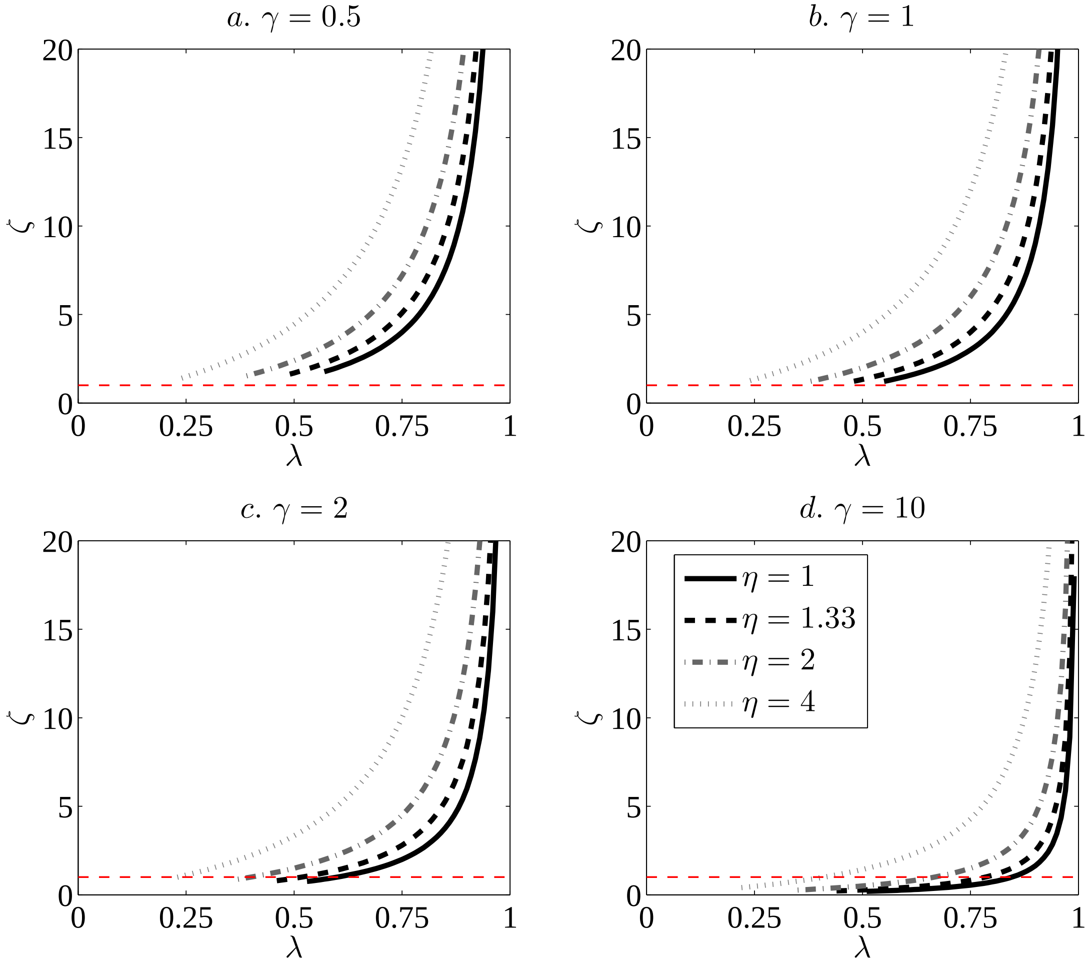


Figure 9: Exposure of consumption of savers to the technology shocks.

Panels $a - d$ plot the parameter values for ζ in the equation linking the consumption of savers to the total output given by (41). Three structural parameters are varied. First, the coefficient of relative risk aversion γ takes values between 0.5 and 10. Each panel represents different values of γ. Second, the Frisch elasticity of labor supply is varied between 0.25 and 1 which implies parameter values for η between 1 and 4. Third, the share of rule-of-thumb consumers λ varies between 0 and 1 (x-axis). All plotted values of ζ are obtained for $\delta < 0$.

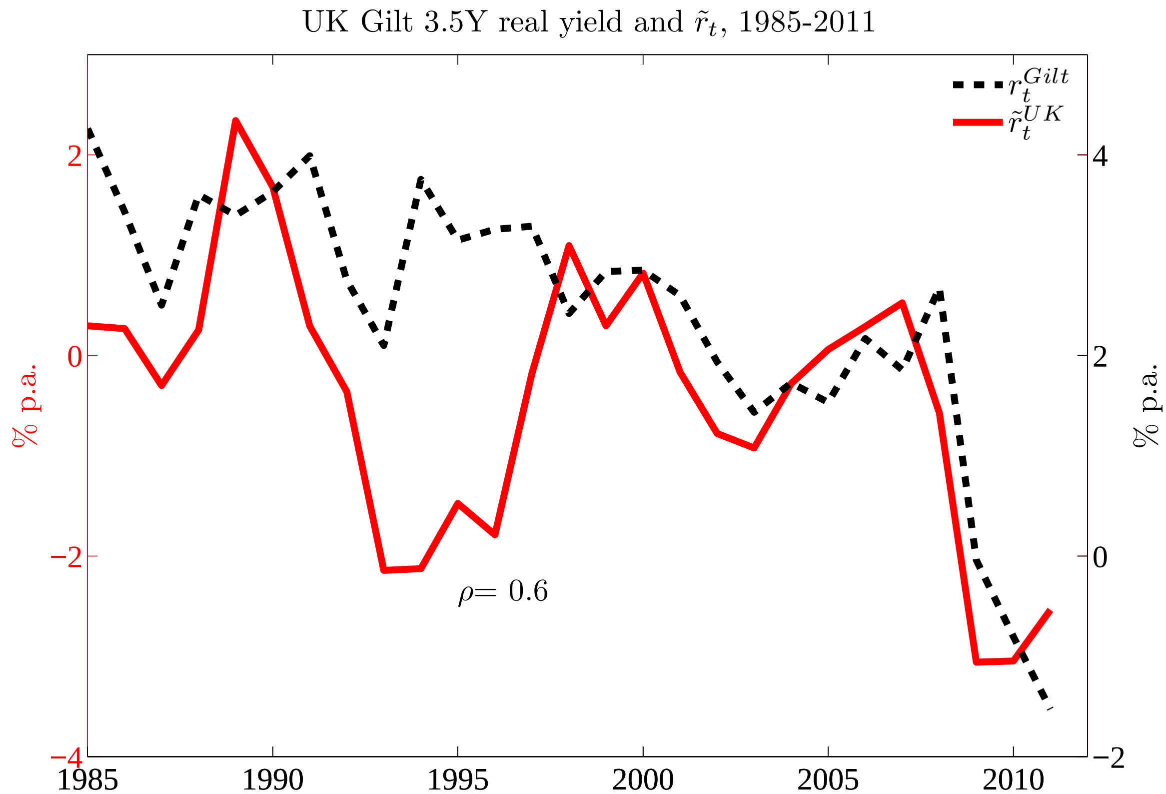


Figure 10: Real yield from UK inflation-indexed government bonds and $\tilde{r}_t$, 1985-2011

The figure plots the proxy for the ex-ante real rate $\tilde{r}_t$ which is computed as the difference between the one- and ten-year nominal Gilt. Superimposed is the real yield with maturity 3.5 years denoted by r_t^{Gilt}. Shorter maturities are not continuously available for real yields in the UK. The data are annual. The sample period is 1985 through 2011.

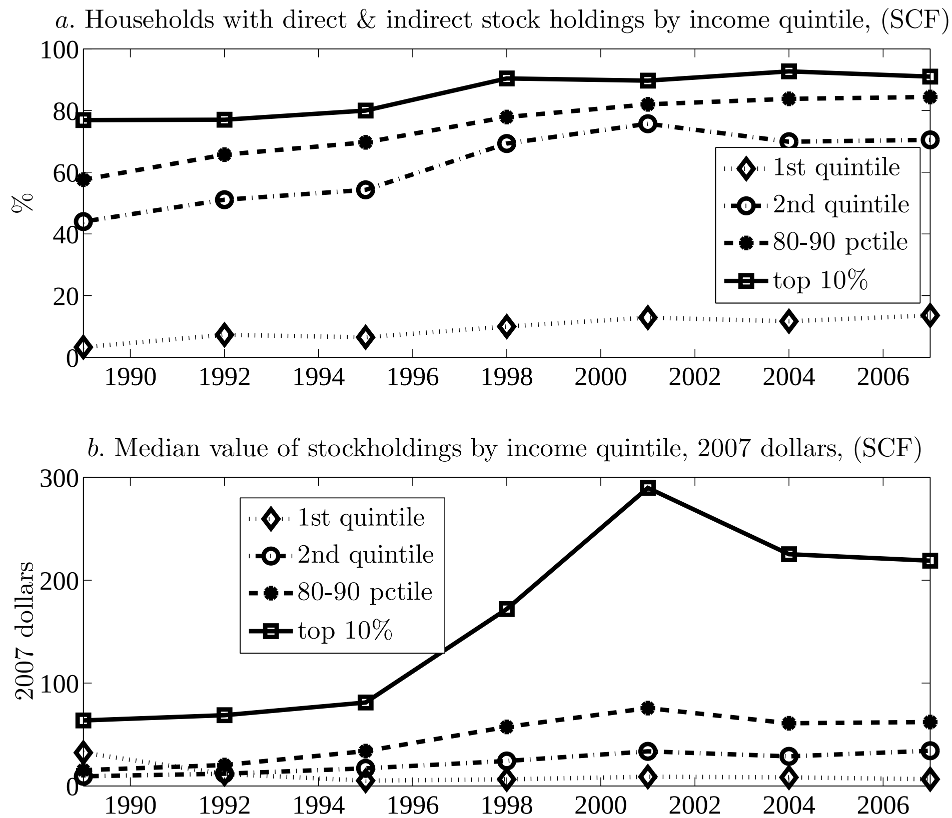


Figure 11: Stock market participation and holdings by income, 1989-2007

Panel *a* plots the percentiles of households with direct or indirect stock holdings split by their income. Two lower quintiles and two upper deciles are displayed for the period 1989-2007. Panel *b* plots the median value of stock holdings of households by income. The values are in 2007 dollars and thus directly comparable across time. The data are obtained from the Survey of Consumer Finance.