Risk Premia and Volatilities in a Nonlinear Term Structure Model

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Paper summary

- The paper estimates a non-linear term structure model that can fit both yields and corresponding volatilities well
- Non-linearity is introduced through a stochastic weighting of two Gaussian models, a framework originally developed for modeling heterogenous beliefs and survival of agents $\rightarrow$ closed-form bond prices
- The non-linearity can capture unspanned stochastic volatility and unspanned risk premia
- Long sample (1961-2014) that includes the Volcker period

Model setup recap

- Gaussian states $X(t)$:

$$dX(t) = \kappa(\bar{X} - X(t))dt + \Sigma dW(t).$$

- SDF:

$$M(t) = M_0(t) \underbrace{\left(1 + \gamma e^{-\beta' X(t)}\right)}_{\frac{1}{s(t)}},$$

with:

$$\frac{dM_0(t)}{M_0(t)} = -r_0(t)dt - \Lambda_0(t)' dW(t),$$

where $r_0(t)$ and $\Lambda_0(t)$ are affine in $X(t)$.

The role of non-linearity

- Fitting yields and volatility with three Gaussian factors:
  1. PC1-PC3 explain $\sim$ 30% of realized volatility, PC1-PC5 explain $\sim$ 40%
  2. The non-linearity, i.e. four additional parameters in γ and β, needs capture the remaining 60-70% of volatility variation

Maturity	PC_1	PC_1-PC_2	PC_1-PC_3	PC_1-PC_4	PC_1-PC_5
Panel A: R^2 for data (realized variance, monthly observations, 1961-2014)					
τ =1	24.3	26.8	35.0	35.7	40.2
τ =2	23.2	24.8	33.7	35.4	41.6
τ =3	21.9	22.8	32.6	35.8	42.5
τ =4	20.3	20.7	31.1	35.9	42.6
τ =5	18.8	18.9	29.6	36.0	42.6

Figure: Panel A Table 5

- Provide more evidence that fitting volatility does not require additional factors

Expected returns in a Gaussian versus non-linear model

- Expected returns in the non-linear model are given by:

$$e(t, T) = \Lambda(t)\sigma(t, T), \tag{1}$$

it would be useful to see the relative contribution of the non-linear part to the variation in market price of risk (MPR) $\Lambda(t)$ and the amount of risk $\sigma(t, T)$.

- The MPR reads:

$$\Lambda(t) = \underbrace{\lambda_{0,0} + \lambda_{0,X} X(t)}_{\text{Gaussian}} + \underbrace{(1 - s(t))\Sigma'\beta}_{\text{non-linear}}. \tag{2}$$

- Note that $\lambda_{0,X} = \lambda_{1,X}$ which leads to (2)

Expected returns in a Gaussian versus non-linear model (cont'd)

- Contribution of non-linearity in volatility:

$$\sigma(T-t) = \omega(t,T)\sigma_0(T-t) + (1-\omega(t,T))\sigma_1(T-t) + \beta(s(t)-\omega(t,T)), \quad (3)$$

  where $\sigma_i = \Sigma' B_i(T-t)$ with Σ identity matrix and $\omega(t,T) = \frac{P_0(t,T)s(t)}{P(t,T)}$

- $\frac{dB_0(\tau)}{d\tau}$ differs from $\frac{dB_1(\tau)}{d\tau}$ by $\rho_{0,0} \Leftrightarrow \rho_{1,0}$, which is mainly driven by β
- To understand the effect of non-linearity, need more intuition on the following:
  1. By how much does $\omega(t,T)$ differ from $s(t)$?
  2. Is $\sigma_0(T-t)$ much different from $\sigma_1(T-t)$?

Model specification checks

- Use long-term bonds (10-year and longer) and the corresponding volatilities to re-estimate the model. Does $s(t)$ change substantially? Which parameters change?
- How would the results (e.g. unspanning) change in different sub-samples, e.g. pre- versus post-Volcker period? Parameter estimates reported for sub-samples are not very informative.
- Using parameter estimates for the full sample and the post-Volcker period, simulate $s(t)$ and compare its properties in two periods:

Simulated moments of $s(t)$ across samples

	Full sample	Post-Volcker
Mean	0.99	0.91
Variance	0.004	0.032

- Referring to (2), simulated $s(t)$ indicates that the role of non-linearity for MPR is minimal

Simulated sample: 640 monthly observations, N=2000

Evaluation of yield and volatility fit

- Authors should provide a more formal assessment of yield and volatility fit from the non-linear model and the $A_1(3)$ model, i.e. report RMSE and compare to other models (e.g. $A_2(4)$)
- Interest rate volatility is quite fast mean-reverting (monthly AR1 coeff. ~ 0.44) $\rightarrow$ study the distribution at shorter horizons, e.g. 1 week through 6 months
- How well does the model match the unconditional yield volatilities? The simulation above indicates potential issues

Broader questions

What are the next big issues in term structure modeling?

- We have reduced-form models that can fit both yields and volatilities, e.g. Cieslak and Povala (2014), Creal and Wu (2014)
- We would like to understand the economic drivers of risk premia and volatility: institutional features, e.g. Haddad and Sraer (2015) or macro fluctuations, e.g. Creal and Wu (2015) $\rightarrow$ can you provide more insights about the equilibrium models?
- Provide a broader motivation for why this a more natural reduced-form setup than others in the literature, e.g. Ghysels, Le, Park, Zhu (2014)

References

1. Cieslak, A., Povala, P. (2014): Information in the term structure of yield curve volatility, Journal of Finance, forthcoming
2. Creal, D., Wu, C. (2014): Interest Rate Uncertainty and Economic Fluctuations, Working paper, Chicago Booth
3. Creal, D., Wu, C. (2015): Bond Risk Premia in Consumption-based Models, Working paper, Chicago Booth
4. Ghysels, E., Le, A., Park, S., Zhu, H. (2014): Risk and Return Trade-Off in the U.S. Treasury Market, Working paper, UNC
5. Haddad, V., Sraer, D. (2015): The Banking View of Bond Risk Premia, Working paper, Princeton and UC Berkeley