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Expected Returns in Treasury Bonds

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We study risk premium in U.S. Treasury bonds. We decompose Treasury yields into inflation expectations and maturity-specific interest-rate cycles, which we define as variation in yields orthogonal to expected inflation. The short-maturity cycle captures the real short-rate dynamics. Jointly with expected inflation, it comprises the expectations hypothesis (EH) term in the yield curve. Controlling for the EH term, we extract a measure of risk-premium variation from yields. The risk-premium factor forecasts excess bond returns in and out of sample and subsumes the common bond return predictor obtained as a linear combination of forward rates. (*JEL* E43, G12)

We study the time variation in the risk premium that investors require for holding Treasury bonds. We propose a novel way of decomposing the nominal yield curve into the risk-premium component and the expectations hypothesis (EH) term (i.e., the average expected short-term interest rate that investor expect to prevail during the life of a bond). Specifically, we decompose Treasury yields into inflation expectations and maturity-specific interest-rate cycles, which we define as the variation in yields that is orthogonal to expected inflation. The short-maturity cycle captures the dynamics of the real short rate at the business-cycle frequency. Jointly with expected inflation, it comprises the EH term in the yield curve. Controlling for the EH term, allows us to extract a measure of the Treasury risk premium from yields. This intuition underlies our construction of the bond return forecasting factor, which we label as the cycle factor and which is our empirical proxy for the time-varying risk premium in Treasuries. The cycle factor is uncorrelated with short-rate expectations; it predicts returns

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on Treasury bonds across the entire maturity spectrum and constitutes the least persistent source of variation in the yield curve. Further, it implies that risk premiums in Treasury bonds vary at a frequency higher than the business-cycle frequency.

Our results are related to the bond return predictor commonly used in the literature—the Cochrane-Piazzesi factor. Cochrane and Piazzesi (2005, CP) demonstrate that a single linear combination of forward rates forecasts bond excess returns at different maturities. We show that the cycle factor subsumes and improves upon the predictability obtained with the linear combination of yields or forward rates. Our approach allows us to predict bond excess returns not only in sample but also out of sample, and thus it can be used in real-time analysis. At the same time, it avoids the known statistical problems that arise when using linear combinations of multiple forward rates, such as the sensitivity to a small measurement error.

The economic basis for our decomposition is the premise that the short-term nominal interest rate is (to a very good approximation) a sum of expected inflation and the real short rate. Building on a large body of literature, we note that highly persistent expected inflation dynamics, often referred to as trend inflation, determines the level of interest rates in the long run and across maturities (e.g., Kozicki and Tinsley 2001; Rudebusch and Wu 2008; Bekaert, Cho, and Moreno 2010). We rely on a simple real-time measure of trend inflation—a discounted moving average (DMA) of past core inflation—which reflects that people update their inflation expectations sluggishly over time. We show that this variable indeed forecasts future inflation well, especially at horizons above one year. The one-period cycle, in turn, encapsulates the mean-reverting part of the short rate around trend inflation and is related to the fluctuations in the real rate. We document that it forecasts changes in the future short rate several years ahead. Neither trend inflation nor the one-period cycle have predictive power for future bond excess returns, and thus, they represent the variation in expectations of the short rate that is independent of the risk premium.

These results allow us to summarize the dynamics of the yield curve across maturities with three observable factors. Trend inflation accounts for about 85% of unconditional variance of yields, determining their overall level. The one-period cycle (real factor) captures more than 60% of the movements in the yield curve slope (i.e., the difference between the long- and the short-term interest rate). Finally, although the cycle factor constitutes the smallest portion of yields' variance, its contribution increases with the maturity, consistent with the intuition that long-maturity yields are most exposed to the fluctuations in the risk premium. Jointly, those three variables explain more than 99% of yield curve variation across maturities. Whereas most modern term-structure models describe yields in terms of principal components (level, slope, and curvature), our decomposition characterizes the yield curve in terms of variables that have a direct link to economic quantities: expected inflation, the real rate, and the risk premium.

We study whether our findings present a robust feature of Treasury yields. We show that our predictability results are not spuriously driven by the well-known biases in the EH tests that arise when working with highly persistent regressors (Bekaert, Hodrick, and Marshall 1997). Nor are they specific to the DMA with which we measure trend inflation. Using survey-based expectations of inflation from a variety of sources, we consistently find that controlling for expected inflation in predictive regressions of bond excess returns strengthens the return predictability relative to the yields-only approach. We document that our way of controlling for the EH term diminishes the sensitivity of the risk premium estimates to the measurement error in yields. Although most of our analysis relies on data starting in 1971, when bonds with maturities of 10 years and above become available, we also reproduce our main conclusions over a long sample period, 1952–2011, using the longest existing inflation survey from Livingston, as well as the Fama–Bliss zero-coupon yields with maturities of 1 through 5 years. Additionally, we obtain direct survey-based measures of the risk premium using professional forecasts of interest rates and document their strong positive correlation with the cycle factor.

1. Illustrative Term-Structure Model

The yield of an n-period nominal Treasury bond can be expressed as the average future short rate $y_t^{(1)}$ expected over the life of the bond, which we refer to as the expectations hypothesis (EH) term, plus the term premium, $rpy_t^{(n)}$:

$$y_t^{(n)} = \underbrace{\frac{1}{n} E_t \sum_{i=0}^{n-1} y_{t+i}^{(1)}}_{\text{EH term}} + rpy_t^{(n)}. \tag{1}$$

The term premium is the sum of expected future excess bond returns, $rpy_t^{(n)} = \frac{1}{n} E_t \left(\sum_{i=0}^{n-2} rx_{t+i+1}^{(n-i)} \right)$, where $E_t(rx_{t+1}^{(n)})$ is the expected one-period excess return, or the risk premium, on an n-period bond, and

$$rx_{t+1}^{(n)} = -(n-1) y_{t+1}^{(n-1)} + n y_t^{(n)} - y_t^{(1)}. \tag{2}$$

Our goal is to decompose the yield curve into the EH component and the term premium in order to study the variation in the Treasury risk compensation, and relatedly, the predictability of bond excess returns.

We illustrate the intuition behind our approach with a stylized term-structure model, which, despite its simplicity, captures the key properties of yield curve dynamics that we document in the data that follows. We assume that nominal yields are driven by three state variables: trend inflation τ_t, the real factor r_t, and the price-of-risk factor x_t. Trend inflation follows a first-order autoregressive process:

$$\tau_t = \mu_\tau + \phi_\tau \tau_{t-1} + \sigma_\tau \varepsilon_t^\tau. \tag{3}$$

Realized inflation π_{t+1} is given by trend inflation plus noise: $\pi_{t+1}=\tau_t+\varepsilon^{\pi}_{t+1}$. Thus, $\tau_t=E_t(\pi_{t+1})$ is expected inflation. We assume that ε^{π}_{t+1} is uncorrelated with other shocks in the economy, implying that the one-period nominal bond is riskless.[1] The nominal one-period interest rate is a function of τ_t and r_t:

$$y_t^{(1)}=\delta_0+\delta_\tau\tau_t+\delta_r r_t, \tag{4}$$

with $\delta_\tau>0, \delta_r>0$, where r_t evolves as:

$$r_t=\mu_r+\phi_r r_{t-1}+\sigma_r\varepsilon_t^r. \tag{5}$$

The real factor r_t can be interpreted as capturing the variation in the real short rate that is independent of the trend inflation, with $\varepsilon_t^\tau, \varepsilon_t^r$ uncorrelated with each other. If $\delta_0=0, \delta_\tau=\delta_r=1$, then r_t equals the *ex-ante* real rate, $r_t^{ex}:=y_t^{(1)}-\tau_t$. If $\delta_\tau>1$ and $\delta_r=1$, as when the Fed reacts more than one for one to expected inflation (e.g., Clarida, Galí, and Gertler 2000), the *ex-ante* real rate $r_t^{ex}=(\delta_\tau-1)\tau_t+r_t$, loads on both τ_t and r_t. Notice that in this setting, the EH component in the yield curve is entirely determined by the trend inflation and the real rate factors. The price-of-risk factor follows an independent autoregressive process:

$$x_t=\mu_x+\phi_x x_{t-1}+\sigma_x\varepsilon_t^x. \tag{6}$$

Letting $F_t=(\tau_t,r_t,x_t)'$, we can write the dynamics of the state and the short rate compactly as:

$$F_t=\mu+\Phi F_{t-1}+\Sigma\varepsilon_t \tag{7}$$

$$y_t^{(1)}=\delta_0+\delta_1' F_t, \tag{8}$$

with Φ and Σ diagonal, $\delta_1=(\delta_\tau,\delta_r,0)'$, and $\varepsilon_t=\left(\varepsilon_t^\tau,\varepsilon_t^r,\varepsilon_t^x\right)'$. Bonds are priced by no arbitrage with the log of the nominal stochastic discount factor:

$$m_{t+1}=-y_t^{(1)}-0.5\Lambda_t'\Lambda_t-\Lambda_t'\varepsilon_{t+1}, \tag{9}$$

where Λ_t is the compensation investors require for facing shocks ε_{t+1}:

$$\Lambda_t=\Sigma^{-1}(\lambda_0+\Lambda_1 F_t). \tag{10}$$

Thus, the parameters of the risk-neutral dynamics used for pricing bonds are:

$$\mu^q=\mu-\lambda_0, \qquad \Phi^q=\Phi-\Lambda_1. \tag{11}$$

Log bond prices $p_t^{(n)}$ and yields $y_t^{(n)}$ are affine in the state vector, $y_t^{(n)}=-\frac{1}{n}p_t^{(n)}$:

$$p_t^{(n)}=\mathcal{A}_n+\mathcal{B}_n' F_t, \quad \text{and} \quad y_t^{(n)}=A_n+B_n' F_t, \tag{12}$$

$$A_n=-\frac{1}{n}\mathcal{A}_n, \quad B_n=-\frac{1}{n}\mathcal{B}_n, \tag{13}$$

where coefficients $\mathcal{A}_n$ and $\mathcal{B}_n$ have standard recursive solutions as a function of $\mu^q, \Phi^q, \Sigma, \delta_0$, and δ_1 (e.g., Duffee 2012), with $\mathcal{A}_1=-\delta_0$ and $\mathcal{B}_1=-\delta_1$.

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[1] The literature on the short-run Fisher hypothesis concludes that the role of term premium variation in short-term nominal yields is negligible (e.g. Shome, Smith, and Pinkerton 1988; Evans and Wachtel 1992; Crowder and Hoffman 1996).

Suppose that investors require compensation for facing shocks to trend inflation and the real factor, and that bond risk premiums at all maturities vary with a single factor x_t, whose own shocks are not priced. This implies:

$$\lambda_0 = \begin{pmatrix} \lambda_{0\tau} \\ \lambda_{0r} \\ 0 \end{pmatrix}, \quad \Lambda_1 = \begin{pmatrix} 0 & 0 & \lambda_{\tau x} \\ 0 & 0 & \lambda_{rx} \\ 0 & 0 & 0 \end{pmatrix}. \tag{14}$$

Specification in Equation (14) mirrors the empirical finding of Cochrane and Piazzesi (2005) that bond risk premiums move on a single mean-revering factor that is largely unexplained by the level, slope, and curvature movements described by most term-structure models.[2] The excess bond log return earned over one period is:

$$rx_{t+1}^{(n)} = \underbrace{\mathcal{B}_{n-1}'(\lambda_0 + \Lambda_1 \mathbf{1}_3)x_t - 0.5\mathcal{B}_{n-1}'\Sigma\Sigma'\mathcal{B}_{n-1}}_{E_t(rx_{t+1}^{(n)})} + \mathcal{B}_{n-1}'\Sigma\varepsilon_{t+1}, \tag{15}$$

where $\mathbf{1}_3$ is 3×1 vector of ones. The first two terms on the right-hand side of Equation (15) describe the risk premium, $E_t(rx_{t+1}^{(n)})$, earned in compensation for ε_{t+1}^{τ} and ε_{t+1}^{r} shocks, plus a convexity adjustment. Note that, with a single factor driving market prices of risk, term premium $rpy_t^{(n)}$ defined in Equation (1) is perfectly correlated with the bond risk premium $E_t(rx_{t+1}^{(n)})$.

The dynamics in Equations (3)–(6) along with specification in Equation (14) lead to tractable expressions for loadings of log bond prices $p_t^{(n)}$ on the state vector F_t:

$$\mathcal{B}_n^{\tau} = -\delta_{\tau}\frac{1-\phi_{\tau}^n}{1-\phi_{\tau}}, \quad \mathcal{B}_n^{r} = -\delta_{r}\frac{1-\phi_{r}^n}{1-\phi_{r}}$$

$$\mathcal{B}_n^{x} = -\mathcal{B}_{n-1}^{\tau}\lambda_{\tau x} - \mathcal{B}_{n-1}^{r}\lambda_{rx} + \mathcal{B}_{n-1}^{x}\phi_x, \tag{16}$$

and yields $y_t^{(n)}$:

$$(B_n^{\tau}, B_n^{r}, B_n^{x})' = -\frac{1}{n}(\mathcal{B}_n^{\tau}, \mathcal{B}_n^{r}, \mathcal{B}_n^{x})', \tag{17}$$

which allows us to understand how different factors affect the yield curve across maturities.

This illustrative setting accommodates several features of inflation and interest rate dynamics documented in the literature. Suppose that the state variables differ in their relative persistence, with trend inflation τ_t being more

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[2] Models with a single source of variation in the price of risk and only a subset of shocks that are priced have been studied, for instance, by Cochrane and Piazzesi (2008), Lettau and Wachter (2011), Campbell, Sunderam, and Viceira (2011), Ang, Bekaert, and Wei (2008). In these models, the price of risk is assumed to follow a univariate process that is independent of other state variables, except in Cochrane and Piazzesi (2008) where it is part of an unrestricted VAR.

persistent as compared with the real factor r_t. A highly persistent trend inflation is consistent with the properties of expected inflation in surveys, as well as with the estimates in the literature (Kozicki and Tinsley 2001; Rudebusch and Wu 2008; Bekaert, Cho, and Moreno 2010). With $\phi_\tau < 1$ but close to unity, loadings of yields on τ_t, B_n^τ, decline slowly with maturity n, generating a level effect in the yield curve. A lower persistence of r_t than that of τ_t lines up with the evidence that variation in the real rate has the most pronounced contribution to yields at short maturities (Ang, Bekaert, and Wei 2008; Fama 1990; Ireland 1996). Indeed, if $\phi_r < \phi_\tau$, then B_n^r declines with n faster than B_n^τ does, thus affecting the slope of the yield curve. Finally, the effect of the price-of-risk factor x_t depends on the signs and magnitudes of $\lambda_{\tau x}$ and λ_{rx} and the persistence of x_t, $\phi_x > 0$. One can expect that a positive shock to trend inflation increases the conditional bond risk premium on nominal bonds in Equation (15), suggesting that $\lambda_{\tau x} < 0$ (because $\mathcal{B}_n^\tau < 0$). The sign of λ_{rx} is intuitively harder to assess because the properties of the real rate remain debated (e.g., Campbell 2006). In the restrictive case, where $\lambda_{\tau x} < 0$ and $\lambda_{rx} = 0$, loadings B_n^x increase with the maturity, and the effect of the risk premium variation is the largest for the long-maturity bonds.

It is useful to define, within the model, the component of the nominal yield curve that is orthogonal to trend inflation:

$$c_t^{(n)} = B_n^r r_t + B_n^x x_t, \tag{18}$$

which we refer to as the "cycle." The composition of $c_t^{(n)}$ changes with maturity. From Equation (4), the one-period cycle, $c_t^{(1)} = \delta_r r_t$, reflects the variation in the real factor r_t but not in the price of risk, x_t. For economically plausible price-of-risk parameters, however, the relative contribution of the price-of-risk factor to the cycles' variance can be expected to increase with maturity. We verify this in subsequent sections when we take the intuition from the model to the data.

2. Data and Methodology

2.1 Data sources

We use end-of-month constant maturity Treasury (CMT) yields from the H.15 statistical release of the Federal Reserve, from November 1971 to December 2011, from which we bootstrap the zero-coupon yield curve. The CMT data comprise maturities of 6 months and 1, 2, 3, 5, 7, 10 and 20 years, which allows us to incorporate long maturities into the analysis. Our sample starts when bonds with maturities of 10 years and above become available.[3]

[3] We bootstrap the zero-coupon yield curve by treating the CMT yields as par yields (see the Treasury website for details on the CMT yields). In the Online Appendix, we compare our realized excess bond returns with other data sets used in the literature (Fama-Bliss data and Gürkaynak, Sack, and Wright (2006, GSW)) showing correlation above 0.99 for returns at corresponding maturities. Our empirical results are robust to the splining procedure used to construct zero-coupon yields, as shown in the Online Appendix.

Inflation is from the FRED database. We use Consumer Price Index (CPI) inflation which is not subject to revisions (Croushore and Stark 1999), and thus can be treated as real-time data. To account for the publication lag (inflation for a given month is released in the middle of the following month), we use the CPI data that are publicly available at the month's end.

2.2 Measuring trend inflation

We rely on a simple measure of trend inflation which, together with the assumption that the nominal short rate carries no risk premium, allows us to identify factors driving the short rate. In particular, we construct a DMA of past CPI inflation:

$$\tau_t^{CPI} = (1-v)\sum_{i=0}^{t-1} v^i \pi_{t-i}, \tag{19}$$

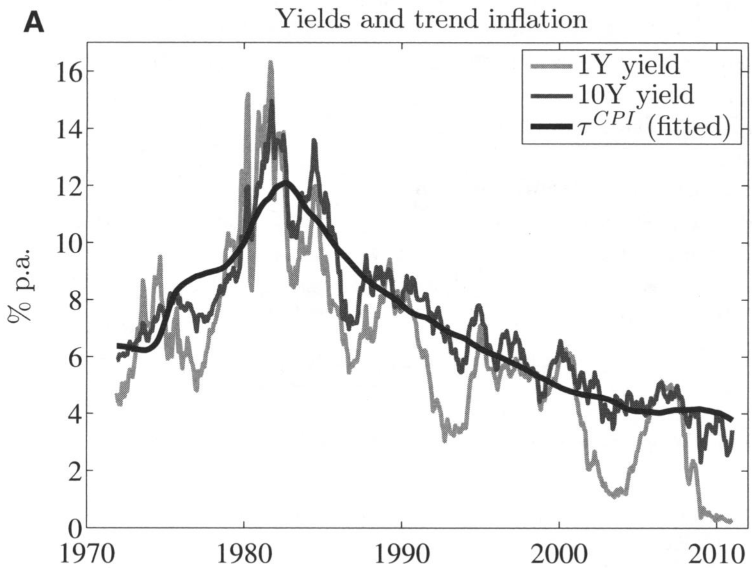
with the year-over-year inflation, $\pi_t = \ln \frac{CPI_t}{CPI_{t-1}}$, and monthly sampling. This approach follows the literature using constant-gain learning to explain macroeconomic dynamics, and inflation dynamics in particular (e.g., Sargent 1999; Kozicki and Tinsley 2001, 2005; Orphanides and Williams 2005; Branch and Evans 2006). Indeed, Equation (19) can be expressed as a constant-gain recursion, $\tau_t^{CPI} = \tau_{t-1}^{CPI} + (1-v)(\pi_t - \tau_{t-1}^{CPI})$, capturing the idea that agents update inflation sluggishly over time.[4] We truncate the sum in Equation (19) at $N = 120$ months and calibrate the v parameters to inflation survey data at $v = 0.987$ (in monthly updating terms), which is well within the range of values between 0.974 to 0.995 considered in the literature. A sensitivity analysis shows that varying N between 100 and 150 months and v between 0.975 and 0.995 leads to very small quantitative differences in our subsequent results. Calibration details, robustness checks, and review of gain parameters used by various authors are contained in the Online Appendix. The Appendix also shows that our results do not depend on using year-over-year, quarterly or monthly inflation.

For the main part of our analysis, we use the core CPI to construct τ_t^{CPI} because core inflation is the measure that bond investors and monetary policy makers focus on (e.g., Blinder and Reis 2005; Mishkin 2007).[5] As an additional check, we verify that our conclusions remain robust to all-items CPI in Section 6.1.

Figure 1, Panel A, superimposes the 1- and 10-year yield with τ_t^{CPI}, showing that τ_t^{CPI} coincides with the low-frequency movement in interest

[4] Constant gain learning can be shown to provide maximally robust optimal prediction rule in the presence of structural breaks and when investors are uncertain about the true data generating process, as is likely the case when forecasting inflation (Evans, Honkapohja, and Williams 2010).

[5] The FRB website reads, "(...) policymakers examine a variety of 'core' inflation measures to help identify inflation trends. (...) Although food and energy make up an important part of the budget for most households—and policymakers ultimately seek to stabilize overall consumer prices—core inflation measures that leave out items with volatile prices can be useful in assessing inflation trends."



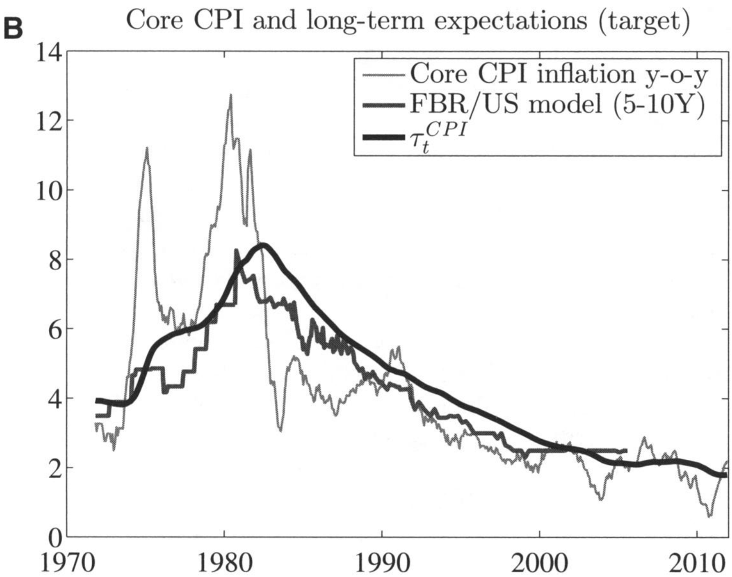


Figure 1
Measuring trend inflation, τ_t^{CPI}

Panel A superimposes the 1- and 10-year yield with the DMA of past core CPI inflation, τ_t^{CPI}. τ_t^{CPI} is fitted to the average level of yields across maturities (slope coefficient of 1.28). Panel B plots the realized year-on-year core CPI inflation together with τ_t^{CPI}, and the long-term (5–10 years ahead) inflation expectations used in the Federal Reserve Board's FRB/US model.

rates. Panel B indicates that τ_t^{CPI} comoves closely with inflation expectations (perceived inflation target) underlying the Federal Reserve Board's FRB/US model (Brayton and Tinsley 1996; Clark and Nakata 2008).[6] Expectations in the FRB/US model are compiled from several sources with irregular frequency and need to be interpolated to a monthly frequency. The advantage of Equation (19) is its simplicity and that it can be constructed in real time.

2.3 Identifying interest-rate cycles

Turning to the role of trend inflation in the cross section of yields, Panel A of Table 1 reports the estimates from projecting yields with different maturities on τ_t^{CPI}:

$$y_t^{(n)} = a_n + b_n^{\tau} \tau_t^{CPI} + \varepsilon_t. \tag{20}$$

The single variable explains between 71% and 89% of yield variance, and its explanatory power is stronger at long maturities. The regression coefficient b_n^{τ} is 1.43 (t-stat = 8.6) at the 1-year maturity and 1.13 (t-stat = 14.9) at the 20-year maturity, indicating that trend inflation drives the level of the yield curve. The loading above one for the 1-year yield aligns with the estimates of the forward-looking Taylor rules in the literature, suggesting that the Fed responds more than one for one to expected inflation (e.g., Coibion and Gorodnichenko 2011; Orphanides 2004).

We define the residual from Regression (20) as the maturity-specific cycle:

$$c_t^{(n)} = y_t^{(n)} - \widehat{a}_n - \widehat{b}_n^{\tau} \tau_t^{CPI}, \tag{21}$$

(i.e., the component of a yield with given maturity that is orthogonal to trend inflation). Notice that $c_t^{(1)}$ encapsulates empirically the variation in the real factor r_t in our model from Section 1. We refer to such residuals as "cycles," to indicate that they are less persistent than yields are. Panel B of Table 1 summarizes their properties. The 1-year cycle $c_t^{(1)}$ is most persistent with a half-life of about 15 months,[7] compared with the ten-year cycle $c_t^{(10)}$ whose half-life is less than a year. At the same time, the half-lives of yield levels are all above 5 years. Cycles of different maturities are relatively highly correlated with each other but less so than yields are. The lowest correlation is 0.55 between $c_t^{(1)}$ and $c_t^{(20)}$, whereas it is 0.89 between $y_t^{(1)}$ and $y_t^{(20)}$.

2.4 Notation

Our empirical approach relies on predictive regressions of bond excess returns. We focus on 1-year holding-period bond excess returns and relegate the analysis

6 The FRB/US series splices data from two surveys of expected long-run inflation—the Hoey survey of financial market participants from 1981 through 1989 and the Survey of Professional Forecasters from 1990. We thank Sharon Kozicki for sharing the series with us. The sample ends in August 2005.

7 Throughout the paper, the half-life is defined as $\ln(0.5)/\ln(|\psi_z|)$, where ψ_z is the estimated first-order autoregressive coefficient for a given variable z_t.

Table 1
Properties of interest-rate cycles

A. Regressions of yields on τ_t^{CPI}: $y_t^{(n)} = a_n + b_n^{\tau}\tau_t^{CPI} + \varepsilon_t$

	$y_t^{(1)}$	$y_t^{(2)}$	$y_t^{(5)}$	$y_t^{(7)}$	$y_t^{(10)}$	$y_t^{(15)}$	$y_t^{(20)}$
$a_n \times 100$	−0.35	−0.12	0.68	1.09	1.43	1.97	2.51
	(−0.45)	(−0.17)	(1.47)	(2.87)	(4.66)	(7.44)	(8.91)
b_n^{τ}	1.43	1.44	1.37	1.32	1.28	1.20	1.13
	(8.64)	(10.31)	(13.06)	(14.58)	(15.98)	(16.26)	(14.93)
$\bar{R}^2$	0.71	0.77	0.84	0.86	0.88	0.89	0.86

B. Properties of cycles: $c_t^{(n)} = y_t^{(n)} - \widehat{a}_n - \widehat{b}_n^{\tau}\tau_t^{CPI}$

	$c_t^{(1)}$	$c_t^{(2)}$	$c_t^{(5)}$	$c_t^{(7)}$	$c_t^{(10)}$	$c_t^{(15)}$	$c_t^{(20)}$
Correlations							
$c_t^{(1)}$	1.00	·	·	·	·	·	·
$c_t^{(2)}$	0.98	1.00	·	·	·	·	·
$c_t^{(5)}$	0.89	0.95	1.00	·	·	·	·
$c_t^{(7)}$	0.82	0.90	0.99	1.00	·	·	·
$c_t^{(10)}$	0.74	0.83	0.95	0.98	1.00	·	·
$c_t^{(15)}$	0.62	0.72	0.88	0.93	0.98	1.00	·
$c_t^{(20)}$	0.55	0.64	0.80	0.87	0.90	0.96	1.00
St.dev. × 100	1.74	1.50	1.14	1.01	0.88	0.81	0.85
Half-life (months)	15.07	14.00	10.75	9.81	9.34	8.72	8.83
Yields:	$y_t^{(1)}$	$y_t^{(2)}$	$y_t^{(5)}$	$y_t^{(7)}$	$y_t^{(10)}$	$y_t^{(15)}$	$y_t^{(20)}$
St.dev. × 100	3.23	3.13	2.84	2.71	2.59	2.43	2.31
Half-life (months)	67.18	84.35	92.34	97.61	107.75	98.71	79.53

Panel A presents univariate regressions of yields on τ_t^{CPI}, as in Equation (20). T-statistics in parentheses are Newey–West adjusted with 18 lags. Panel B reports unconditional correlations between cycles of different maturities, as well as standard deviations and half-lives of cycles. The half-life is defined as $\ln(0.5)/\ln(|\psi_z|)$, where ψ_z is the estimated first-order autoregressive coefficient for a given variable z_t.

of other holding periods to the Online Appendix. In predictive regressions, we use monthly overlapping data and measure time in years. Thus, $rx_{t+1}^{(n)}$ defined in Equation (2) is a 1-year holding-period excess return on a n-year zero-coupon bond. $p_t^{(n)}$ is the log bond price, $p_t^{(n)} = -ny_t^{(n)}$, and the 1-year forward rate locked in for the time between $t+n-1$ and $t+n$ is $f_t^{(n)} = p_t^{(n-1)} - p_t^{(n)}$. Our sample has 470 monthly observations: the last observation is the return realized in December 2011, and it is predicted with variables observed until December 2010. On several occasions, we take an average of excess returns across maturities, denoted by $\overline{rx}_{t+1}$. Because the volatility of returns scales proportionally with bond duration, we duration standardize returns to avoid overweighting particular maturities (i.e., $\overline{rx}_{t+1} = \frac{1}{19}\sum_{k=2}^{20} rx_{t+1}^{(k)}/k$). Our results are not sensitive to this convention, and working with simple averages would not change our conclusions. The Online Appendix reports descriptive statistics for excess returns. Unless the context is unclear, we do not introduce separate notation for yields $y_t^{(n)}$ and cycles $c_t^{(n)}$ from the affine model in Section 1 and the data.

2.5 Accounting for small-sample biases

Predictive regressions of returns are plagued by small sample biases. We consider these biases in two ways. First, we rely on conservative standard errors from reverse regressions proposed by Hodrick (1992) and extended by Wei and Wright (2013). To remove the overlap in the error term, this approach exploits the covariance of one-period returns with an h-period sum of the predictor ($h=12$ for monthly data and annual returns). Ang and Bekaert (2007) show reverse regressions standard errors to have superior small-sample properties compared with the commonly used Hansen-Hodrick or Newey-West errors, both of which overreject the null hypothesis of no predictability in small samples.[8]

Second, because of the bias stemming from highly persistent interest rates, predictive R^2's from regression-based tests of the EH are known to overstate the true degree of predictability (Bekaert, Hodrick, and Marshall 1997). Therefore, we obtain a small sample distribution of R^2's under the null of EH (i.e., where the true R^2 is zero). Using the model in Section 1, we simulate yields at the monthly frequency ($T=470$ months) and construct annual excess returns under the assumption that risk premiums are zero (i.e., $\lambda_0=0, \Lambda_1=0$). We allow for different levels of persistence of the trend inflation τ_t ($\phi_\tau=\{0.8, 0.975, 0.999\}$) and the real factor r_t ($\phi_r=\{0.6, 0.75, 0.9\}$). We calibrate the other parameters of τ_t and r_t as follows: Using Regression (20) for the 1-year yield ($n=1$), we set $\delta_0=0, \delta_\tau=1.43$ and $\delta_r=1$. For each level of persistence, factor volatilities σ_τ and σ_r are calibrated to match the unconditional standard deviation of τ_t^{CPI} equal to 1.90%, and the unconditional standard deviation of the 1-year cycle, $c_t^{(1)}$, equal to 1.74%, respectively. Using simulated data, we then run predictive regressions of excess returns on trend inflation and the real factor. The 95th quantile of the distribution of adjusted R^2's ($\bar{R}^2$'s) from these regressions ranges from 19% to 23% across different parameter configurations. Thus, even under the EH, one can expect to obtain meaningful predictive $\bar{R}^2$. Details of the $\bar{R}^2$ distributions are reported in the bottom panel of Table 2, and we use them as a benchmark to evaluate the $\bar{R}^2$ obtained in the data in the next section.

3. Main Empirical Result

3.1 Predictive regressions

This section presents predictive regressions of bond excess returns. The estimates are collected in Table 2. We use the average return $\overline{rx}_{t+1}$ as the left-hand-side variable and report results for specific maturities in subsequent tables.

[8] Wei and Wright (2013) extend the reverse regressions proposed by Hodrick (1992) beyond testing the null hypothesis of no predictability and allowing for highly persistent regressors. We follow their reverse regression delta method, which they show to deliver superior results in situations when return predictability can be large. The 1-month excess returns on bonds, needed for this procedure, are obtained as in Campbell and Shiller (1991), by approximating the log price of a $(n-1/12)$-maturity bond as $-(n-1/12)y_t^{(n)}$.

Consider the standard forecasting regression of excess returns on multiple yields, which we call the yields-only regression (Column (1) of Table 2):

$$\overline{rx}_{t+1} = d_0 + \sum_i d_i y_t^{(i)} + \varepsilon_{t+1} \tag{22}$$

$$\text{p-value Wald test} = 0.05; \quad \bar{R}^2 = 0.24.$$

Yield maturities used as regressors are 1, 2, 5, 7, 10 and 20 years. Estimating Regression (22) on forward rates, as in Cochrane and Piazzesi (2005, 2008), or adding other maturities gives essentially identical results. The Wald test for the joint significance of the regression coefficients rejects the EH at the 5% level, and the regression $\bar{R}^2$ is 0.24. The results are similar if we regress $\overline{rx}_{t+1}$ on the average yield $\bar{y}_t = \frac{1}{19}\sum_{i=2}^{20} y_t^{(i)}$ and the 1-year yield $y_t^{(1)}$, as in Column (3) of Table 2, giving a p-value of 0.04 and an $\bar{R}^2$ of 0.18.

The evidence in favor of time-varying risk premiums strengthens when we augment the yields-only regression with our measure of trend inflation, τ_t^{CPI}. We label this regression as yields-plus-τ_t^{CPI} (Column (2) of Table 2):

$$\overline{rx}_{t+1} = d_0 + \sum_i d_i y_t^{(i)} + \underbrace{d_\tau}_{\substack{-1.02 \\ (-4.30)}} \tau_t^{CPI} + \varepsilon_{t+1} \tag{23}$$

$$\text{p-value Wald test} = 0.00; \quad \bar{R}^2 = 0.54.$$

The variable τ_t^{CPI} is highly statistically significant, and the $\bar{R}^2$ doubles compared with the yields-only case.

Concerns that arise in regressions using multiple yields are overfitting and multicollinearity. Indicative of the latter, in Columns (1)–(2) of Table 2 coefficients are jointly highly significant but insignificant individually. However, it turns out that one can simplify Regression (23) without losing the predictive power:

$$\overline{rx}_{t+1} = d_0 + \underbrace{d_1}_{\substack{1.45 \\ (5.03)}} \bar{y}_t + \underbrace{d_2}_{\substack{-0.61 \\ (-3.70)}} y_t^{(1)} + \underbrace{d_\tau}_{\substack{-1.01 \\ (-4.65)}} \tau_t^{CPI} + \varepsilon_{t+1} \tag{24}$$

$$\text{p-value Wald test} = 0.00; \quad \bar{R}^2 = 0.53.$$

Importantly, the entire predictive content of the regression comes from a component in yields that is orthogonal to trend inflation τ_t^{CPI}. To see this, consider predicting returns with the cycles defined in Equation (21):

$$\overline{rx}_{t+1} = \bar{\gamma}_0 + \underbrace{\gamma_1}_{\substack{1.45 \\ (5.03)}} \bar{c}_t + \underbrace{\gamma_2}_{\substack{-0.61 \\ (-3.67)}} c_t^{(1)} + \varepsilon_{t+1} \tag{25}$$

$$\text{p-value Wald test} = 0.00; \quad \bar{R}^2 = 0.53,$$

where $\bar{c}_t = \frac{1}{19}\sum_{i=2}^{20} c_t^{(i)}$. The predictive power remains unchanged compared with (24).

Table 2
Predictive regressions

A. Predictive regressions

Regressors →	Yields only (1)	Yields+τ^{CPI} (2)	$\bar{y}_t, y_t^{(1)}$ (3)	$\bar{y}_t, y_t^{(1)}, \tau_t^{CPI}$ (4)	$\bar{c}_t, c_t^{(1)}$ (5)
Regression coefficients					
$y^{(1)}$ or $c^{(1)}$	−1.13	−1.09	−0.42	−0.61	−0.61
	(−1.87)	(−1.64)	(−2.48)	(−3.70)	(−3.67)
$y^{(2)}$ or $c^{(2)}$	0.73	1.06	–	–	–
	(0.62)	(0.81)	–	–	–
$y^{(5)}$ or $c^{(5)}$	0.83	−0.71	–	–	–
	(0.99)	(−0.10)	–	–	–
$y^{(7)}$ or $c^{(7)}$	0.40	0.51	–	–	–
	(0.15)	(0.32)	–	–	–
$y^{(10)}$ or $c^{(10)}$	−1.15	0.84	–	–	–
	(−1.69)	(0.43)	–	–	–
$y^{(20)}$ or $c^{(20)}$	0.37	0.21	–	–	–
	(0.94)	(0.49)	–	–	–
τ^{CPI}	–	−1.02	–	−1.01	–
	–	(−4.30)	–	(−4.65)	–
$\bar{y}$ or $\bar{c}$	–	–	0.54	1.45	1.45
	–	–	(2.47)	(5.03)	(5.03)
Regression statistics					
$\bar{R}^2$	0.24	0.54	0.18	0.53	0.53
Wald test	12.34	34.86	6.46	28.61	25.34
pval	0.05	0.00	0.04	0.00	0.00
Rel.prob. (BIC)	0	3e-4	0	0.57	1.00

B. Distribution of predictive $\bar{R}^2$ under EH, $T=470$ months

	$\phi_r=0.75$			$\phi_\tau=0.975$		
	$\phi_\tau=0.8$	$\phi_\tau=0.975$	$\phi_\tau=0.999$	$\phi_r=0.6$	$\phi_r=0.75$	$\phi_r=0.9$
P5	0.00	0.01	0.01	0.01	0.01	0.01
P95	0.19	0.23	0.20	0.22	0.22	0.23

In panel A, the LHS variable is a duration-standardized excess bond return averaged across maturities, $\overline{rx}_{t+1}$. Columns (1) through (5) use different regressors: (1) six yields; (2) same yields as in (1) plus trend inflation τ_t^{CPI}; (3) two yield variables: $y_t^{(1)}$ and $\bar{y}_t$; (4) $y_t^{(1)}$ and $\bar{y}_t$ plus τ_t^{CPI}; (5) two cycle variables: $c_t^{(1)}$ and the average cycle $\bar{c}_t$. T-statistics for individual coefficients, the Wald test and the corresponding p-values are obtained using the reverse regression delta method. Row labeled "Rel.prob. (BIC)", where BIC is the Bayesian information criterion, gives the relative probability of a model i computed as $\exp\{(BIC_{\text{best}} - BIC_i)T/2\}$, where $BIC = \ln(\widehat{\sigma^2}) + \ln(T)n/T$, n is the number of regressors, $\widehat{\sigma^2} = SSE/T$ of the regression, and T is the sample size. Relative probability of one indicates the best model selected by a given criterion. Relative probability of zero means that a given model has zero probability to explain the data equally well as the best model. Panel B reports the 5th and 95th percentiles of the $\bar{R}^2$ obtained under the null of EH from 10,000 Monte Carlo simulations of the model in Section 1. The parameters are $\delta_0=0, \delta_\tau=1.43, \delta_r=1$, and σ_τ, σ_r are calibrated to match st.dev.$(\tau_t)=1.90\%$ and st.dev.$(r_t)=1.74\%$ at each level of persistence of ϕ_τ, ϕ_r.

The $\bar{R}^2$ from the yields-plus-τ_t^{CPI} or cycles regressions increase visibly relative to those obtained only with yields. However, these numbers should be compared to the $\bar{R}^2$'s that arise under the null of the EH (Panel B of Table 2). Except for the regression in Column (3), all specifications produce $\bar{R}^2$'s that are above the maximum 95th quantile under EH (0.23), albeit the yields-only approach exceeds this threshold by only a small margin.

The $\bar{R}^2$'s of all specifications that include τ_t^{CPI} lie well above the EH benchmark.[9]

To assess the relative merit of the different regression specifications, Table 2 contains relative probabilities based on the Bayesian information criterion (BIC).[10] A relative probability of one indicates the best model selected by the criterion, and a relative probability of zero means that a model has zero chance of explaining the data equally well as the best model does. The BIC clearly favors models that condition on τ_t^{CPI}, and in particular parsimonious specifications in Equations (24) and (25) that do not rely on multiple yields. We therefore explore such specifications in more detail in the next sections.

3.2 Trends

An important question is whether our results would arise if we used moving averages (MA) of past interest rates rather than τ_t^{CPI} to detrend yields. In Panel A of Table 3, we replace τ_t^{CPI} in Equation (24) with a smoothed past interest rate, denoted τ_t^{yld}. We construct τ_t^{yld} in several variants: using either 1- or 5-year yield, and different smoothing methods: the DMA as in Equation (19), or a simple MA with window sizes of 12, 24, or 60 months. We consistently find that τ_t^{yld} adds no predictive content to the regressions, and we cannot reject that the coefficient on τ_t^{yld} is zero. This result extends to the case in which we include τ_t^{yld} jointly with τ_t^{CPI}. For instance, using the DMA of the 1-year yield as τ_t^{yld}, we have the following:

$$\overline{rx}_{t+1} = d_0 + \underbrace{d_1}_{\substack{-0.58\\(-3.51)}} y_t^{(1)} + \underbrace{d_2}_{\substack{1.39\\(4.37)}} \bar{y}_t + \underbrace{d_3}_{\substack{0.11\\(0.75)}} \tau_t^{yld} + \underbrace{d_4}_{\substack{-1.09\\(-4.81)}} \tau_t^{CPI} + \varepsilon_{t+1}. \tag{26}$$

Panel B of Table 3 presents analogous regressions for the alternative ways of constructing τ_t^{yld}. The loading on τ_t^{yld} remains statistically insignificant across all specifications, whereas the loadings and the significance of the other three variables are nearly unchanged compared with Regression (24).

The Online Appendix contains additional robustness checks that indicate that these results do not depend on the use of the DMA for inflation (which can be replaced with a simple MA), or the particular choices of the smoothing windows for the interest rate. Likewise, they hold true if we use the T-bill rate rather than 1-year yield, and for individual bond returns rather than $\overline{rx}_{t+1}$. Additionally, the Online Appendix investigates whether our results could spuriously arise under entirely random trends that have nothing to do with inflation or the yield curve.

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9 The Online Appendix shows that this conclusion is robust to varying the gain parameter v and the moving window N over which we compute the DMA for τ_t^{CPI} in Equation (19).

10 The relative probability for model i relative to the best model selected by BIC is $RelPr_i = \exp\{(BIC_{\text{best}} - BIC_i)T/2\}$, where $BIC = \ln(\widehat{\sigma^2}) + \ln(T)m/T$, m is the number of regressors, $\widehat{\sigma^2} = SSE/T$ of the regression, and T is the sample size.

Table 3
Predictive regressions with trends based on smoothed past interest rates

$\tau_t^{yld} \rightarrow$	Smoothed 1-year yield, $y_t^{(1)}$				Smoothed 5-year yield, $y_t^{(5)}$			
	DMA (1)	MA_{12} (2)	MA_{24} (3)	MA_{60} (4)	DMA (5)	MA_{12} (6)	MA_{24} (7)	MA_{60} (8)
A. Smoothed past interest rates: $\overline{rx}_{t+1} = d_0 + d_1 y_t^{(1)} + d_2 \bar{y}_t + d_3 \tau_t^{yld} + \varepsilon_{t+1}$								
$y^{(1)}$	−0.48	−0.42	−0.43	−0.51	−0.47	−0.42	−0.47	−0.51
	(−2.94)	(−2.19)	(−2.26)	(−3.20)	(−2.84)	(−2.51)	(−2.98)	(−3.25)
$\bar{y}_t$	0.72	0.54	0.53	0.75	0.66	0.56	0.76	0.78
	(2.52)	(2.38)	(2.28)	(2.56)	(2.38)	(2.10)	(2.81)	(2.59)
τ_t^{yld}	−0.16	−0.00	0.01	−0.16	−0.11	−0.02	−0.18	−0.18
	(−0.94)	(0.03)	(0.09)	(−0.88)	(−0.70)	(−0.11)	(−0.99)	(−0.93)
$\bar{R}^2$	0.20	0.18	0.18	0.21	0.19	0.18	0.20	0.21
B. Smoothed past interest rate plus trend inflation: $\overline{rx}_{t+1} = d_0 + d_1 y_t^{(1)} + d_2 \bar{y}_t + d_3 \tau_t^{yld} + d_4 \tau_t^{CPI} + \varepsilon_{t+1}$								
$y^{(1)}$	−0.58	−0.74	−0.73	−0.56	−0.58	−0.64	−0.58	−0.56
	(−3.51)	(−3.55)	(−3.68)	(−3.33)	(−3.46)	(−3.84)	(−3.66)	(−3.34)
$\bar{y}_t$	1.39	1.42	1.35	1.37	1.39	1.17	1.29	1.37
	(4.37)	(5.04)	(4.90)	(4.24)	(4.36)	(4.02)	(4.44)	(4.25)
τ_t^{yld}	0.11	0.21	0.29	0.13	0.09	0.44	0.30	0.14
	(0.75)	·(1.26)	(1.59)	(0.64)	(0.63)	(1.74)	(1.45)	(0.67)
τ_t^{CPI}	−1.09	−1.08	−1.13	−1.11	−1.07	−1.21	−1.24	−1.13
	(−4.81)	(−4.62)	(−4.75)	(−4.51)	(−4.94)	(−4.82)	(−4.40)	(−4.42)
$\bar{R}^2$	0.54	0.55	0.57	0.54	0.54	0.59	0.57	0.54

Table 3 presents predictive regressions of $\overline{rx}_{t+1}$ on $y_t^{(1)}$, $\bar{y}_t$, τ_t^{yld} (smoothed past interest rate), and τ_t^{CPI}. In the columns we consider different ways of smoothing the past interest rates: either 1-year yield (Columns (1)–(4)) or 5-year yield (Columns (5)–(8)). Columns labeled DMA apply the discounted moving average in Equation (19) with the same parameters as for inflation. Columns MA_{12}, MA_{24}, and MA_{60} use smoothed past interest rates with a simple moving average over the window of 12, 24, and 60 months, respectively. Reverse regression t-statistics are in parentheses.

To evaluate this possibility, we set up the null hypothesis that bond returns are predictable with a linear combination of yields, and reproduce our regressions replacing τ_t^{CPI} with simulated driftless random walks. We find that there is less than 0.1% chance to obtain our empirical estimates with random trends. Overall, we conclude that it is very unlikely that our results are driven by biases because of inclusion of a spurious trend.

3.3 Single return-forecasting factor

Cochrane and Piazzesi (2005) show that a single forecasting factor, a fitted value from projecting $\overline{rx}_{t+1}$ on a set of time-t forward rates, captures the variation in expected bond excess returns across different maturities. We propose an alternative way of constructing the single factor. To this end, we use the fitted value from Regression (25), which we label as the cycle factor, $\widehat{cf}_t$:

$$\widehat{cf}_t = \hat{\gamma}_0 + \hat{\gamma}_1 c_t^{(1)} + \hat{\gamma}_2 \bar{c}_t. \tag{27}$$

Alternatively, one could use the fitted value from Regression (24) which is 99.5% correlated with the $\widehat{cf}_t$ variable above, but we choose specification in Equation (25), to highlight the properties of the cycles. The motivation for including the average cycle $\bar{c}_t$ in the construction of $\widehat{cf}_t$ is pragmatic: Similar

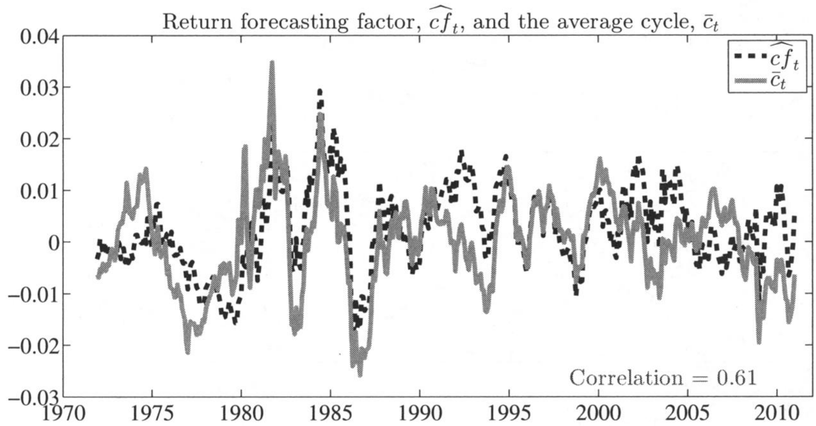

Figure 2
The cycle factor $\widehat{cf}$ and the average cycle, $\bar{c}_t$
Figure 2 shows the time series of the cycle factor $\widehat{cf}_t$ and the average cycle across maturities $\bar{c}_t$.

to the first principal component in yields, $\bar{c}_t$ is a simple way to summarize the cross-sectional information, it attenuates measurement error in yields, and is less arbitrary than using a cycle with a particular maturity. We plot the cycle factor in Figure 2 together with the average cycle $\bar{c}_t$. With a half-life below 10 months (monthly AR(1) coefficient 0.925), $\widehat{cf}_t$ is less persistent than any individual cycle is.

We run regressions of individual excess returns, $rx_{t+1}^{(n)}$, on the cycle factor. Panel A of Table 4 shows that $\widehat{cf}_t$ is a significant return predictor across different maturities. Because one may worry about generated regressors, we bootstrap the reverse regression statistics and provide 5th and 95th percentiles of their distribution using the Li and Maddala (1997) bootstrap. Using the cycle factor, there is essentially no loss in terms of the predictive content compared with the highly parameterized yields-plus-τ_t^{CPI} Regression (23) (row "$\Delta \bar{R}^2$" reports the difference in $\bar{R}^2$). Panel B of Table 4 presents bivariate regressions of $rx_{t+1}^{(n)}$ on $c_t^{(1)}$ and $c_t^{(n)}$, indicating that $\widehat{cf}_t$ summarizes well the risk premium information in the yield curve relative to the maturity-specific cycles. In the Online Appendix, we discuss other approaches to constructing the single forecasting factor that lead to essentially identical risk premium dynamics, and by extension, to the same return predictability.

4. Interpreting the Results

We reveal a simple structure of three orthogonal factors to describe the cross-section of yields: trend inflation τ_t^{CPI}, the one-period cycle $c_t^{(1)}$, and the cycle factor $\widehat{cf}_t$. We show that trend inflation and one-period cycle subsume the EH

Table 4
Predicting returns with the cycle factor

	$rx^{(2)}$	$rx^{(5)}$	$rx^{(7)}$	$rx^{(10)}$	$rx^{(15)}$	$rx^{(20)}$
A. Cycle factor						
$rx_{t+1}^{(n)}=\beta_0+\beta_1\widehat{cf}_t+\varepsilon_{t+1}^{(n)}$, where $\widehat{cf}_t=\hat{\gamma}_0+\hat{\gamma}_1c_t^{(1)}+\hat{\gamma}_2\bar{c}_t$						
$\widehat{cf}_t$	0.62	0.68	0.70	0.73	0.74	0.72
	(3.80)	(4.64)	(4.92)	(5.16)	(5.26)	(5.06)
t-stat (SS,[5%,95%])	[1.31,4.27]	[2.05,4.91]	[2.38,5.16]	[2.56,5.38]	[2.72,5.47]	[2.66, 5.30]
$\bar{R}^2$	0.38	0.46	0.49	0.53	0.54	0.51
$\Delta\bar{R}^2$	0.02	0.01	0.01	0.01	0.01	0.04
B. Maturity-specific cycles						
B1. $rx_{t+1}^{(n)}=\alpha_0+\alpha_1c_t^{(1)}+\alpha_2c_t^{(n)}+\varepsilon_{t+1}$						
$c_t^{(1)}$	−1.60	−0.93	−0.72	−0.54	−0.39	−0.31
	(−2.82)	(−3.67)	(−3.77)	(−3.71)	(−3.07)	(−2.55)
$c_t^{(n)}$	1.95	1.63	1.52	1.43	1.27	1.11
	(3.26)	(4.73)	(5.16)	(5.29)	(5.10)	(4.88)
$\bar{R}^2$	0.33	0.46	0.51	0.53	0.52	0.50
B2. $rx_{t+1}^{(n)}=\alpha_0+\alpha_1c_t^{(n)}+\varepsilon_{t+1}$						
$c_t^{(n)}$	0.13	0.36	0.49	0.63	0.76	0.76
	(2.24)	(2.94)	(3.34)	(3.57)	(3.77)	(3.88)
$\bar{R}^2$	0.04	0.11	0.16	0.23	0.30	0.34

Panel A shows the predictability of individual bond excess returns achieved with the cycle factor. The cycle factor $\widehat{cf}_t$ is defined in Equation (27). The row denoted "t-stat (SS,[5%,95%])" summarizes the small sample distributions of the reverse regression t-statistics obtained with nonparametric block bootstrap. Row $\Delta\bar{R}^2$ reports the difference in $\bar{R}^2$ between yields-plus-τ_t^{CPI} Regression (23) and the single-factor regression. Panel B presents regressions of individual excess returns on cycles of a given maturity. T-statistics in parentheses are obtained with the reverse regression delta method.

term in yields. We also map our decomposition back into the illustrative term structure model.

4.1 Expectations hypothesis terms

Regression (24) uses three variables to forecast bond returns, all of which are highly significant. To understand the role of these variables in predicting returns, let us consider restricting the loading on $\bar{y}_t$ to zero:

$$\overline{rx}_{t+1}=d_0+\underbrace{d_1}_{\substack{0.002\\(0.51)}}y_t^{(1)}+0\times\bar{y}_t+\underbrace{d_\tau}_{\substack{-0.04\\(-0.54)}}\tau_t^{CPI}+\varepsilon_{t+1} \tag{28}$$

$$\text{p-value Wald test}=0.84;\quad \bar{R}^2=0.01.$$

Excluding $\bar{y}_t$ changes the regression estimates diametrically. The coefficients on $y_t^{(1)}$ and τ_t^{CPI} are effectively zero; the predictive $\bar{R}^2$ drops down to 0.01; and the p-value from the Wald joint significance test exceeds 0.8. Therefore, because our trend-inflation proxy and the 1-year yield alone contain no predictive power for bond excess returns, they must control the EH component embedded in $\bar{y}_t$ that is orthogonal to the variation in the risk premium. It is important to recognize that our initial specification in Equation (24) does not impose such orthogonality upfront because, in general, the risk premium and the EH components could load on the same set of factors. In a related way, the one-period cycle $c_t^{(1)}$ alone does not predict returns:

$$\overline{rx}_{t+1}=\gamma_0+\underbrace{\gamma_1}_{\substack{0.002\\(0.53)}} c_t^{(1)}+\varepsilon_{t+1}, \quad \bar{R}^2=0.00. \tag{29}$$

Thus, in Regression (25), $c_t^{(1)}$ controls for the EH term that is orthogonal both to trend inflation and to the risk premium.

4.1.1 Predicting short-rate changes. If τ_t^{CPI} and $y_t^{(1)}$ capture the EH term in the yield curve, they should predict future short rates. Table 5 reports forecasts of changes in the 1-year yield, $\Delta y_{t,t+h}^{(1)}$, at horizons h from 1 through 4 years ahead, as studied in the literature following Fama and Bliss (1987). To set a benchmark for our results, Panel A of Table 5 reports the standard Fama and Bliss (1987) regressions of yields changes $\Delta y_{t,t+h}^{(1)}$ on the forward-spot spread $f_t^{(n)}-y_t^{(1)}$, showing that the predictive power of the spread is overall weak. Fama (2006) revisits this evidence by arguing that the short rate mean-reverts toward a time-varying rather than a constant mean. He proposes to predict short-rate changes with a deviation of the 1-year yield from a 60-month MA of the 1-year yield, as well as with a dummy variable that takes a value of one up to August 1981 and zero afterwards. Such regressions are reported in Panel B of Table 5, with and without the dummy. Without the dummy, including the MA of the 1-year yield, $\tau_t^{yld}(y^{(1)},\mathrm{MA}_{60})$, does not change the conclusion relative to the Fama-Bliss specification: The coefficients on $\tau_t^{yld}(y^{(1)},\mathrm{MA}_{60})$ are insignificant at all forecast horizons, and the amount of variance explained is nearly the same as in Panel A. This parallels our previous finding that a MA of past yields does not contain predictive power for future bonds returns (Section 3.2). In line with Fama (2006), the dummy variable, denoted as D_{81}, is key for the empirical success of his regressions (Panel B).

Although the D_{81} dummy is chosen based on hindsight knowledge of the turning point in yields in the early 1980s, the τ_t^{CPI} proxy identifies this point in real time, linking the long-run mean of yields to trend inflation. Panel C of Table 5 predicts short-rate changes with $y_t^{(1)}$ and τ_t^{CPI}. The coefficients are jointly highly significant at all horizons (with p-values not exceeding 0.02). A negative coefficient on $y_t^{(1)}$ and a positive coefficient on τ_t^{CPI} indicate that a low short rate relative to trend inflation signals that the short rate will rise in the future. We also present univariate regressions of yield changes $\Delta y_{t,t+h}^{(1)}$ on the 1-period cycle, $c_t^{(1)}$: The cycle predicts short-rate changes with a negative coefficient ranging from -0.40 at the 1-year horizon to -1.37 at the 4-year horizon, and an R^2 from 0.15 to 0.47. This supports the interpretation of the one-period cycle as the mean-reverting component of the short rate. Indeed, if the short rate followed a univariate AR(1) process, the slope coefficient

Table 5
Predicting short-rate changes

A. Fama-Bliss (1987) regressions
$\Delta y^{(1)}_{t,t+h} = \alpha_0 + \alpha_1 (f^{(h)}_t - y^{(1)}_t) + \varepsilon_{t,t+h}$

h years	1	2	3	4
const.$\times$100	−0.18	−0.75	−1.61	−2.06
	(−0.31)	(−0.63)	(−1.02)	(−1.13)
$f^{(h)}_t - y^{(1)}_t$	0.11	0.56	1.09	1.20
	(0.18)	(0.67)	(1.44)	(1.85)
$\bar{R}^2$	0.00	0.05	0.19	0.25

B. Fama (2006) regressions
$\Delta y^{(1)}_{t,t+h} = \alpha_0 + \alpha_1 y^{(1)}_t + \alpha_2 \tau^{yld}_t (y^{(1)}, \mathrm{MA}_{60}) + \alpha_3 D_{81} + \varepsilon_{t,t+h}$

	Without 1981 dummy				With 1981 dummy			
h years	1	2	3	4	1	2	3	4
const.$\times$100	0.44	0.64	0.74	0.90	−0.38	−0.83	−1.32	−1.60
	(0.67)	(0.54)	(0.45)	(0.44)	(−0.71)	(−0.80)	(−0.89)	(−0.85)
$D_{81}\times 100$	–	–	–	–	2.41	4.09	5.57	6.62
	–	–	–	–	(3.06)	(2.71)	(2.25)	(1.87)
$y^{(1)}_t$	−0.15	−0.42	−0.63	−0.77	−0.41	−0.83	−1.16	−1.38
	(−0.77)	(−1.32)	(−1.75)	(−2.16)	(−1.83)	(−2.32)	(−2.72)	(−3.01)
$\tau^{yld}_t (y^{(1)}, \mathrm{MA}_{60})$	0.05	0.25	0.42	0.51	0.33	0.70	1.01	1.20
	(0.26)	(0.69)	(0.82)	(0.76)	(1.65)	(2.05)	(2.31)	(2.40)
$\bar{R}^2$	0.04	0.13	0.19	0.24	0.27	0.43	0.61	0.73
pval Wald	0.69	0.41	0.21	0.09	0.01	0.00	0.00	0.01

C. Trend inflation and 1-period cycle

	$\Delta y^{(1)}_{t,t+h} = \alpha_0 + \alpha_1 y^{(1)}_t + \alpha_2 \tau^{CPI}_t + \varepsilon_{t,t+h}$				$\Delta y^{(1)}_{t,t+h} = \alpha_0 + \alpha_1 c^{(1)}_t + \varepsilon_{t,t+h}$			
h years	1	2	3	4	1	2	3	4
const.$\times$100	−0.25	−0.39	−0.31	0.04	−0.12	−0.24	−0.36	−0.45
	(−0.39)	(−0.33)	(−0.19)	(0.02)	(−0.41)	(−0.42)	(−0.42)	(−0.41)
$y^{(1)}_t$	−0.40	−0.90	−1.25	−1.39	–	–	–	–
	(−2.08)	(−2.87)	(−3.49)	(−3.91)	–	–	–	–
τ^{CPI}_t	0.57	1.25	1.68	1.78	–	–	–	–
	(2.80)	(3.41)	(3.30)	(2.66)	–	–	–	–
$c^{(1)}_t$	–	–	–	–	−0.40	−0.90	−1.24	−1.37
	–	–	–	–	(−2.09)	(−2.87)	(−3.47)	(−3.84)
$\bar{R}^2$	0.15	0.34	0.46	0.48	0.15	0.34	0.46	0.47
pval Wald	0.02	0.00	0.00	0.00	–	–	–	–

Table 5 presents the predictability of future changes in the 1-year yield, $\Delta y^{(1)}_{t,t+h} = y^{(1)}_{t+h} - y^{(1)}_t$, for horizons h from 1 to 4 years ahead. In Panel B (without dummy) and C, rows "pval Wald" report p-values for a Wald test that regression coefficients (excluding constant) are jointly equal to zero; the p-value in panel B (with dummy) is for a Wald test that all regression coefficients (including constant) are zero. The dummy variable D_{81} in panel B takes value of one up till August 1981 and zero afterwards following Fama (2006). T-statistics (in parentheses) and Wald tests are obtained with reverse regressions.

in the regression of $\Delta y^{(1)}_{t,t+h}$ on $y^{(1)}_t$ would approach -1 and the R^2 would approach 0.5 as the horizon h increases (see e.g., the Appendix in Fama and Bliss 1987). Our evidence is consistent with the mean reversion of the short rate toward slowly moving trend inflation rather than constant mean. The univariate regression with $c^{(1)}_t$ has essentially the same predictive content as the bivariate specification with τ^{CPI}_t and $y^{(1)}_t$, implying that there is a significant variation in short-rate expectations at the business cycle frequency that is uncorrelated with the trend inflation.

Overall, these results support the interpretation that τ_t^{CPI} and $c_t^{(1)}$ capture the EH component of the yield curve. In the Online Appendix, we forecast short-rate changes with τ_t^{CPI} and $c_t^{(1)}$ out of sample, showing that these two variables outperform alternative predictors of short-rate changes, such as random walk or a model using the principal components of yields as predictors.

4.1.2 Predicting inflation. The DMA of past core CPI inflation, τ_t^{CPI}, is not selected to predict best either short-rate changes or future excess returns, but rather it is constructed as a measure of expected inflation. Therefore, the second important element of our interpretation is that τ_t^{CPI} is indeed a good proxy for inflation expectations. To show that this is so, we use today's value of τ_t^{CPI} to forecasts quarterly inflation $\pi_{t+h/4}^q = 4\ln\frac{CPI_{t+h/4}}{CPI_{t+(h-1)/4}}$, $h=\{1,4,8,12,16\}$. For consistency with the literature (e.g., Ang, Bekaert, and Wei 2007; Faust and Wright 2013), we form out-of-sample forecasts using nonoverlapping quarterly observations. The data are sampled in the middle month of each quarter.[11] We compare the forecasts obtained with τ_t^{CPI} with the following models:

Random walk. We consider two random walk models to forecast $\pi_{t+h/4}^q$—a simple random walk (SRW) that uses π_t^q as predictor, and an annual random walk (AORW) that uses $\frac{1}{4}\sum_{j=0}^{3}\pi_{t-j/4}^q$. AORW follows Atkeson and Ohanian (2001).

Survey-based forecasts. Short-term forecasts (LivST6M, LivST12M) for 6 and 12 months ahead are from the Livingston survey; long-term forecasts (LivLT) are compiled from Livingston and Blue Chip Economic Indicators (BCEI) surveys by the Philadelphia Fed and are available from 1979:Q4. Surveys are conducted semiannually. Thus, if a survey is available in March, our first quarterly forecast is for inflation between March and June.

ARMA(1,1). An ARMA(1,1) model is estimated recursively with data up to time t.

We form out-of-sample forecasts starting in 1979:Q4, 1984:Q4, and 1995:Q1, and the last forecast is made for 2011:Q4. Following Ang, Bekaert, and Wei (2007), we use forecasts from the ARMA(1,1) model as a benchmark. Table 6 reports the ratios of RMSEs obtained from different models to the RMSEs from ARMA(1,1). A ratio less than one means that a given model

[11] In a study of inflation forecasting methods, Faust and Wright (2013) construct the DMA of quarter-over-quarter inflation, sampled quarterly with the gain parameter 0.95. Our inflation forecasts are virtually the same if we sample quarterly our τ_t^{CPI} proxy (i.e., constructed with year-over-year inflation, monthly updating, and gain parameter 0.987) or if we follow their approach (see the Online Appendix).

outperforms ARMA(1,1). We provide forecasts for both the core and all-items inflation, but τ_t^{CPI} is always based on core inflation as in our empirical analysis in Section 3. The ARMA(1,1) model is estimated starting in 1967:Q1, which is when the first observation of τ_t^{CPI} becomes available.

The main results can be summarized as follows. The τ_t^{CPI} variable performs well at longer horizons (four quarters and above). For longer horizons and all-items inflation, τ_t^{CPI} delivers the lowest RMSE ratios out of all competing models; for core inflation, the performance of τ_t^{CPI} is comparable to AORW. The long-term survey produces somewhat more accurate forecasts of core inflation, but it underperforms τ_t^{CPI} in forecasting all-items inflation. These results are consistent with Cogley (2002), who shows that constant-gain learning produces more accurate inflation forecasts over the medium run (1 through 3 years) compared with alternative models. Likewise, Faust and Wright (2013) stress the importance of accounting for the slow-moving component of inflation in forecasting. We notice that although with either τ_t^{CPI} or long-term surveys we use one forecast without differentiating across horizons, they generally outperform short-term surveys. This fact supports our use of one trend-inflation proxy as a measure of inflation expectations across all yield maturities.

4.2 Interpretation of interest-rate cycles

Our results up to this point suggest that cycles with different maturities do not move on a single factor (see e.g., their unconditional correlation matrix in Table 1). It is therefore useful to understand the factor structure that underlies the cross-section of cycles. Panel B2 of Table 4 displays the slope coefficients and the R^2 from univariate regressions of $rx_{t+1}^{(n)}$ on $c_t^{(n)}$ for different n. The predictive content of the cycles for future returns increases with the maturity, but the predictability is significantly weaker than it is in bivariate regressions that include both $c_t^{(1)}$ and $c_t^{(n)}$ (Panel B1). This is because cycles contain not only the term premium but also the EH component, which we have shown to be uncorrelated with the premium. The relative importance of the latter decreases with the maturity, as visible in the increasing R^2 in univariate regressions. However, even at long maturities, cycles are not direct measures of the risk premium—a fact that shows in the significance of $c_t^{(1)}$ at long maturities.

In terms of economic interpretation, one would expect the one-period cycle $c_t^{(1)}$ to be linked to the *ex-ante* real rate (i.e., $r_t^{ex} = y_t^{(1)} - E_t(\pi_{t+1})$). Figure 3 superimposes $c_t^{(1)}$ with two measures of the *ex-ante* real rate constructed using 1-year-ahead expected inflation from the Livingston survey and the Survey of Professional Forecasters (SPF). The correlation between the survey-based *ex-ante* real rate and $c_t^{(1)}$ is 0.59 and 0.63 for the two surveys, respectively. Although both share very similar business cycle variation, the survey-based *ex-ante* real rate displays a more persistent behavior. Intuitively, if the loading of the short rate on expected inflation is higher than one, as suggested by our

Table 6
Predicting inflation

	A. Core CPI							B. All items CPI					
Horizon (qtrs)	(1) SRW	(2) AORW	(3) τ^{CPI}	(4) LivLT 10Y	(5) LivST 12M	(6) LivST 6M	Horizon (qtrs)	(1) SRW	(2) AORW	(3) τ^{CPI}	(4) LivLT 10Y	(5) LivST 12M	(6) LivST 6M
Start of out-of-sample period: Q4 1979													
$h=1$	1.09	0.93	1.25	1.15	1.14	1.34	$h=1$	1.19	1.01	1.11	0.88	0.93	1.00
$h=4$	1.05	0.95	1.08	0.73	1.01	1.24	$h=4$	1.25	0.99	1.04	1.12	1.15	1.27
$h=8$	1.07	0.99	0.93	0.70	1.09	1.36	$h=8$	1.17	1.06	0.97	1.06	1.15	1.30
$h=12$	1.04	0.96	0.87	0.59	1.04	1.30	$h=12$	1.15	1.02	0.94	0.99	1.12	1.25
$h=16$	0.94	0.88	0.82	0.56	0.90	1.15	$h=16$	1.09	1.03	0.92	0.98	1.11	1.24
Start of out-of-sample period: Q4 1984													
$h=1$	1.11	0.88	1.32	1.04	1.15	1.43	$h=1$	1.22	1.02	1.00	0.71	0.84	0.89
$h=4$	0.80	0.69	0.93	0.70	0.75	0.98	$h=4$	1.27	0.94	0.95	1.17	1.13	1.19
$h=8$	0.62	0.57	0.70	0.55	0.63	0.81	$h=8$	1.14	0.97	0.90	1.16	1.17	1.25
$h=12$	0.58	0.51	0.60	0.49	0.58	0.72	$h=12$	1.09	0.86	0.85	1.09	1.08	1.11
$h=16$	0.53	0.47	0.57	0.45	0.54	0.66	$h=16$	0.98	0.85	0.81	1.04	1.03	1.08
Start of out-of-sample period: Q1 1995													
$h=1$	1.05	0.91	1.08	1.14	1.16	1.51	$h=1$	1.23	1.02	0.89	0.68	0.70	0.72
$h=4$	0.80	0.69	0.74	0.76	0.83	1.07	$h=4$	1.33	0.96	0.88	1.23	1.25	1.30
$h=8$	0.59	0.55	0.56	0.56	0.65	0.84	$h=8$	1.15	0.99	0.88	1.24	1.28	1.36
$h=12$	0.52	0.45	0.46	0.48	0.58	0.72	$h=12$	1.13	0.88	0.86	1.21	1.21	1.23
$h=16$	0.45	0.39	0.42	0.43	0.50	0.60	$h=16$	1.05	0.89	0.82	1.18	1.19	1.22

Table 6 presents ratios of out-of-sample root-mean-squared errors (RMSEs) for forecasts of quarterly inflation. The forecasts are formed for a single quarter inflation at a given horizon, $\pi^q_{t+h/4}$. The table reports the RMSEs from different models relative to the RMSE from ARMA(1,1). The models considered are: (1) simple random walk (SRW), (2) annual random walk (AORW), (3) only τ^{CPI}, and three surveys with different forecast horizons (4) LivLT (long-term inflation forecasts 10 years ahead, compiled by Philadelphia Fed from BCEI and Livingston surveys), (5) LivST 12M (12-month-ahead Livingston forecasts), (6) LivST 6M (6-month-ahead Livingston forecast). Survey forecasts are available semiannually, and therefore we use the data at the survey frequency. The ARMA(1,1) model is estimated at a quarterly frequency starting in Q1 1967. Out-of-sample forecasts are formed starting in Q4 1979, Q4 1984, and Q1 1995 and the last forecast is made for Q4 2011. The quarter in which we start each out-of-sample period is dictated by the timing of survey data.

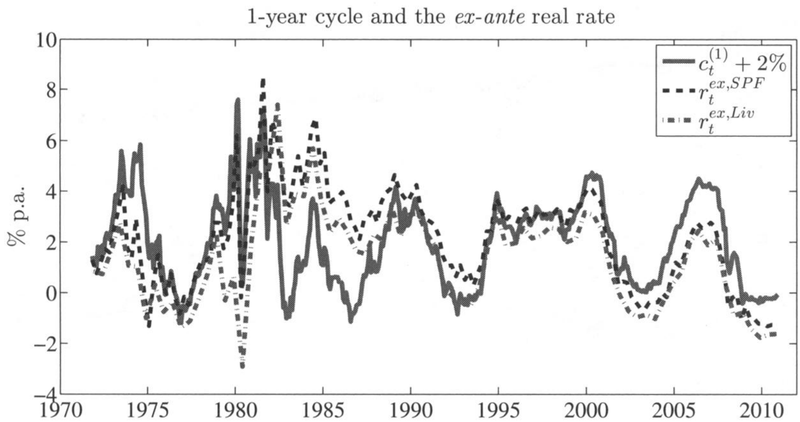

Figure 3
Short-maturity cycle and the *ex-ante* real rate
The figure compares the *ex-ante* real rate with 1-year interest rate cycle, $c_t^{(1)}$. The *ex-ante* real rate is obtained as $r_t^{ex,s} = y_t^{(1)} - E_t^s(\pi_{t+1})$, where $E_t^s(\pi_{t+1})$ is expected inflation 1-year ahead from Livingston $s = Liv$ or SPF $s = SPF$ survey, respectively. For ease of comparison, we add 2% to $c_t^{(1)}$. The Livingston survey is available semiannually, and the SPF survey is quarterly.

estimates (see Panel A in Table 1), then by definition the *ex-ante* real rate r_t^{ex} will reflect part of the trend-inflation dynamics.[12] As such, $c_t^{(1)}$ can be interpreted as the component of the *ex-ante* real rate that is orthogonal to expected inflation.

4.3 Yield curve factors and factor loadings of yields

4.3.1 Regression loadings. The preceding discussion leads to a simple description of the yield curve in terms of three observable variables $\tilde{F}_t = (\tau_t^{CPI}, c_t^{(1)}, \widehat{cf}_t)'$, which we have connected to expected inflation, the real rate, and the risk premium, respectively. We project yields on the factors $\tilde{F}_t$ to illustrate their cross-sectional effect on the yield curve:

$$y_t^{(n)} = \tilde{A}_n + \tilde{B}_n^{\tau} \tau_t^{CPI} + \tilde{B}_n^{r} c_t^{(1)} + \tilde{B}_n^{x} \widehat{cf}_t + e_t^{(n)}. \tag{30}$$

On average, the regression explains 99.7% of the variation in yields across maturities, with the lowest fraction for the 20-year yield (98.8%) and the highest for the 1-year yield (100%, by construction). Figure 4 displays the regression loadings of yields on factors as a function of maturity (solid lines). The loadings are scaled to represent the effect of a one-standard-deviation shock to a factor. As expected, τ_t^{CPI} has a level effect on the yield curve. The loadings of $c_t^{(1)}$ decline with maturity, but are still sizeable even at $n = 20$ years; those of $\widehat{cf}_t$ increase with maturity. This cross-sectional pattern agrees

[12] A regression of $y_t^{(1)}$ on the SPF (Livingston) forecast of inflation $E_t^s(\pi_{t+1})$ gives a slope coefficient of 1.31 (1.10), t-statistic of 5.98 (6.39), and an $\bar{R}^2$ of 67% (65%).

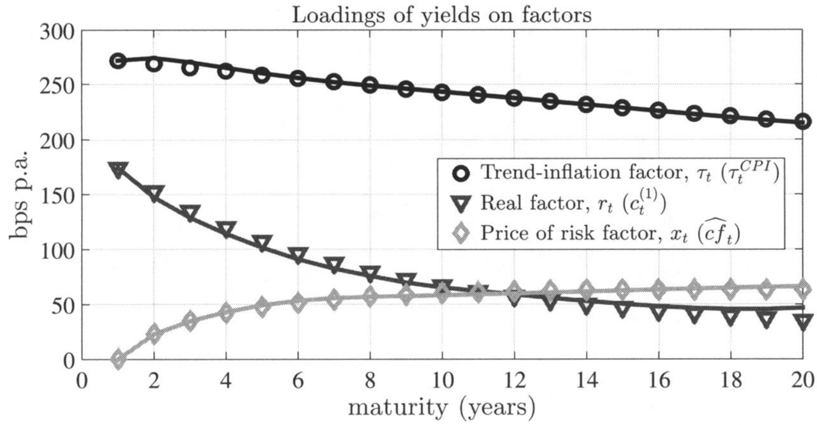

Figure 4
Loadings of yields on factors: regressions vs. affine model
The solid lines present the loadings of yields on observable factors $\tilde{F}_t = \left(\tau_t^{CPI}, c_t^{(1)}, \widehat{cf}_t\right)'$ obtained from Regression (30). The markers present the loadings obtained from the affine model given in Equation (17) for factors $F_t = (\tau_t, r_t, x_t)'$. The parameters of the affine model are calibrated by minimizing the sum of squared distances between the loadings from the regression and from the affine model, see Equations (31)–(32).

with an increasing contribution of the term premium for longer-term cycles, as evidenced in Panel B of Table 4 above. In terms of magnitudes, we find that a one-standard-deviation shock to each of the factors, τ_t^{CPI}, $c_t^{(1)}$ and $\widehat{cf}_t$, moves the yield curve by about 240, 80, and 50 basis points, respectively, on average across maturities. The effect of $c_t^{(1)}$ ($\widehat{cf}_t$) is 175 (0, by construction) basis points at the 1-year maturity and 50 (60) basis points at the 20-year maturity.

A common way to summarize the cross-sectional information in the yield curve is in terms of principal components (PCs): level, slope, and curvature. We find that the level is most strongly related to τ_t^{CPI}, which captures 86.5% of its variance, whereas $\widehat{cf}_t$ and $c_t^{(1)}$ make up less than 4% and 10%, respectively. The slope loads mainly on $c_t^{(1)}$ and $\widehat{cf}_t$, which account for 61% and 32% of its variance, respectively; and τ_t^{CPI} accounts for less than 3%. Given that the 1-period cycle itself does not forecast returns, its large contribution to the slope explains why slope is a noisy measure of the bond risk premium.

4.3.2 Affine model loadings. It is useful to connect these results to the affine model in Section 1 to shed light on the parameters of trend inflation, the real factor, and the price-of-risk factor that are consistent with the cross-sectional loadings in Regression (30). The model-based counterparts of $\tilde{F}_t = (\tau_t^{CPI}, c_t^{(1)}, \widehat{cf}_t)'$ are $F_t = (\tau_t, r_t, x_t)'$. Notice that our observables $\tilde{F}_t$ satisfy the assumption in the model that factors are orthogonal. While this is obviously true for $c_t^{(1)}$ and τ_t^{CPI}, the correlation between $\widehat{cf}_t$ and $c_t^{(1)}$ is less than 0.01, in line with the fact that $c_t^{(1)}$ does not predict bond returns. Using the estimates of Regression (30) at different maturities, we can calibrate the degree of persistence of trend inflation and of the real factor, as well as the market price of risk parameters that would generate such loadings in the affine model. To this end, we minimize the sum of squared distances:

$$\hat{\phi}_\tau = \min_{\phi_\tau} \sum_{i=1}^{20} (B_i^\tau - \tilde{B}_i^\tau)^2; \quad \hat{\phi}_r = \min_{\phi_r} \sum_{i=1}^{20} (B_i^r - \tilde{B}_i^r)^2 \tag{31}$$

$$(\hat{\lambda}_{\tau x}, \hat{\lambda}_{rx}) = \min_{\lambda_{\tau x}, \lambda_{rx}} \sum_{i=1}^{20} (B_i^x - \tilde{B}_i^x)^2, \tag{32}$$

where B_n^τ, B_n^r, B_n^x are defined in Equation (17). We set the values of $\delta_\tau = 1.43$ and $\delta_r = 1$, as discussed in Section 2.5. To calibrate the market price-of-risk parameters in Equation (32), we fix the mean reversion of the price-of-risk factor at $\phi_x = 0.392$, which is estimated from the time series of $\widehat{cf}_t$ (i.e., the monthly AR(1) coefficient of 0.925 raised to the 12th power).[13]

The annual AR(1) coefficients calibrated in Equation (31) are $\hat{\phi}_\tau = 0.975$ and $\hat{\phi}_r = 0.75$. This implies a highly persistent trend-inflation dynamics, with the real factor being the relatively more transitory component of the short rate. For the price of risk parameters, we find $\hat{\lambda}_{\tau x} = -0.47$ and $\hat{\lambda}_{rx} = 0.16$. Economically, these parameters mean that a positive shock to trend inflation (real-rate factor) increases (decreases) the conditional bond risk premium (see Equation (15)). A larger magnitude of $\hat{\lambda}_{\tau x}$ than $\hat{\lambda}_{rx}$ is consistent with the view that shocks to trend inflation (level shocks in the yield curve) are the main source of time-varying risk compensation in the yield curve (e.g., Cochrane and Piazzesi 2008).[14]

The calibrated affine loadings (B_n^τ, B_n^r, B_n^x) are represented by markers in Figure 4 and are plotted against the coefficients from Regression (30). Despite the simplicity of the affine model, these two sets of coefficients line up very closely.

To show that the affine model can match important properties of the yield curve, we obtain the remaining parameters as follows: Factor means and standard deviations are calibrated to match means and standard deviations

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13 We could calibrate ϕ_x jointly with $\lambda_{\tau x}, \lambda_{rx}$ from Equation (32). However, analysis of the B_n^x loadings in Recursion (17) shows that λ's always appear premultiplied by a function of ϕ_x. Thus, in practice, these parameters turn out to be difficult to identify jointly. The advantage of fixing ϕ_x at the AR(1) coefficient estimated from the time series of the cycle factor $\widehat{cf}_t$ is that it leaves our calibration fewer degrees of freedom, and allows us to verify whether the persistence of the risk premiums we estimate from the data is in fact consistent with economically plausible market price of risk parameters.

14 Although there is little disagreement in the literature that the nominal bond premiums are positive, the pricing of real-rate shocks is harder to judge. TIPS are available over a relatively short sample and have been exposed to a significant liquidity premium. However, Barr and Campbell (1997) document that average excess returns on real bonds in the United Kingdom are negative. Moreover, a negative real term premium arises if real rates are procyclical, a fact that finds support in the estimates of Ang, Bekaert, and Wei (2008), and in equilibrium models with recursive preferences such as Piazzesi and Schneider (2006). In a related way, using a long-run risk model, Bansal and Shaliastovich (2013) show that bond risk premium increases with inflation uncertainty and decreases with real uncertainty.

Table 7
Calibration of the affine model

A. Calibrated parameters

	δ_0	δ_τ	δ_r	ϕ_τ	ϕ_r	ϕ_x
	0	1.43	1	0.98	0.75	0.39
	$E(\tau_t)$	$E(r_t)$	$E(x_t)$	Std(τ_t)	Std(r_t)	Std(x_t)
in %	3.50	1.00	0	1.90	1.74	0.81
	μ_τ	μ_r	μ_x	σ_τ	σ_r	σ_x
par$\times$100	0.09	0.25	0	0.42	1.14	0.74
	$\lambda_{0\tau}$	λ_{0r}	$\lambda_{\tau x}$	λ_{rx}		
	−0.002	0.001	−0.47	0.16		

B. Model-implied unconditional moments of yields and excess returns (% p.a.)

	Yields, $y_t^{(n)}$			Excess returns, $rx_{t+1}^{(n)}$		
	$n=1$	$n=5$	$n=10$	$n=2$	$n=5$	$n=10$
Mean	5.99	6.36	6.81	0.18	0.74	1.62
St.dev.	3.23	2.84	2.59	1.36	4.56	9.09

Panel A reports the calibrated parameters of the affine model from Section 1. δ_0, δ_τ, and δ_r are from Regression (20) for $y_t^{(1)}$. ϕ_τ, ϕ_r are calibrated in Equation (31). ϕ_x is estimated as the AR(1) coefficient from the monthly time series of $\widehat{cf}_t$ (monthly AR(1) of 0.925), and raised to the 12th power to obtain the annual mean reversion. μ_τ, μ_r, μ_x and $\sigma_\tau, \sigma_r, \sigma_x$ are calibrated to match means and standard deviations of factors $\tau_t^{CPI}, c_t^{(1)}, \widehat{cf}_t$, respectively. μ_x is fixed at zero for x_t to capture just the conditional variation in risk premium, whereas $\lambda_{0\tau}, \lambda_{0r}$ are calibrated to generate realistic unconditional average excess returns (these parameters are inconsequential for our analysis). $\lambda_{\tau x}$ and λ_{rx} are obtained from Equation (32). Panel B reports the unconditional means and standard deviations of yields and excess returns at the calibrated model parameters.

of $\left(\tau_t^{CPI}, c_t^{(1)}, \widehat{cf}_t\right)'$; μ_x is fixed at zero, whereas $\lambda_{0\tau}$ and λ_{0r} are calibrated to sample means of yields and excess returns (although these parameters are inconsequential for our analysis). All parameters are reported in Table 7 (Panel A). The model generates realistic unconditional moments of yields and excess returns (Panel B).

In sum, three observable factors $\tau_t^{CPI}, c_t^{(1)}, \widehat{cf}_t$ provide a simple novel decomposition of the cross-section of interest rates. This decomposition is supported with the intuition from the affine model about how trend inflation, the real factor, and the market prices of risk affect the Treasury yield curve.

4.4 Measurement error

In an affine model, yields are linear functions of a low number of factors. Thus, in principle, one should be able to recover factors from yields. Using our three-factor affine model from Section 1 as an example, let $\mathbf{y}_t = (y_t^{(n_1)}, y_t^{(n_2)}, y_t^{(n_2)})'$ be a vector of model-implied yields with different maturities, then $\mathbf{y}_t = A + BF_t$, where vector $A_{(3\times 1)}$ and matrix $B_{(3\times 3)}$ contain the affine loadings. If matrix B is of full rank, we can invert the equation to perfectly recover factors from yields: $F_t = B^{-1}(\mathbf{y}_t - A)$. Thus, from the perspective of an affine setting, a natural question that arises is why the additional information we provide about trend inflation helps recover the risk premium in the yield curve more effectively than information contained in yields themselves does.

Observed yields are contaminated by a small measurement error because of bid-ask spreads, and perhaps more importantly, because of the splining of zero-coupon yields. The measurement error resulting from splining is about seven to nine basis points, as shown by Bekaert, Hodrick, and Marshall (1997). Using Fama-Bliss (FB), Gürkanyak-Sack-Wright (GSW), and CMT-based zero-coupon yields, we compute the RMSEs between yields of corresponding maturities in each dataset for our sample period as $\text{RMSE}_{i,j} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(y_t^{(n),i} - y_t^{(n),j})^2}, i \neq j$, where $i, j = \{\text{FB}, \text{GSW}, \text{CMT}\}$. The average, minimum and maximum RMSE is 11, 6, and 20 basis points across datasets and maturities from 1 through 10 years.

We illustrate the effects of measurement error on the ability to recover the risk premium variation from yields within the framework of our affine model. Suppose that yields are observed with an additive noise η_t, which is uncorrelated with true model-based yields, $y_t^{(n)}$, at all leads and lags and is also uncorrelated across maturities:

$$y_t^{(n),\eta} = y_t^{(n)} + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta). \tag{33}$$

From Equation (15), the realized excess returns in the model are driven by the risk premium factor x_t and orthogonal shocks. Thus, estimating the risk premium via predictive regressions of realized excess returns amounts to uncovering the variation in x_t. If yields are observed without measurement error, a regression of x_t on true yields $\mathbf{y}_t$ gives an R^2 equal to one. However, this is no longer true if yields are noisy. To demonstrate this, suppose we want to explain the variation in the price-of-risk factor using the following regression[15]:

$$x_t = \alpha_0 + \alpha_Z' Z_t^\eta + \varepsilon_t^Z, \tag{34}$$

and consider different specifications for Z_t^η. In the yields-only specifications, Z_t^η involves: three yields, six yields, or three PCs of yields. For the yields-plus-τ_t specification, Z_t^η contains two yields and τ_t. Trend inflation can itself be measured with iid noise, $\tau_t^\epsilon = \tau_t + \epsilon_t$, $\epsilon_t \sim N(0, \sigma_\epsilon)$. Measurement error in the regressor leads to the attenuation bias in the least-squares estimates of α_Z, and thus lowers the regression R^2 away from unity. We first evaluate how the bias depends on size of measurement error, σ_η. We simulate 470 monthly observations from the affine model using the parameters calibrated in the previous section. We then run Regressions (34) on simulated data. The simulation is repeated 10,000 times, and we report average R^2 obtained across simulations.

15 To make the results more transparent, we focus on regressions of x_t as the dependent variable rather than rx_{t+1}, which contains x_t and the orthogonal unpredictable shock. This allows us to explore the consequences of the measurement error in explanatory variables. The measurement error in explanatory variables is a far more important source of bias in the least-squares estimates than the measurement error is in the response variable (e.g., Wooldridge 2010).

Panel A of Figure 5 displays the R^2 from Regression (34) as a function of σ_η, which takes values from 0 to 15 basis points. When $\sigma_\eta = 0$, as expected, we recover $R^2 = 1$ for all specifications of Z_t^η, except the six-yields case, which is degenerate in a model with three shocks and thus omitted. However, even a small amount of measurement noise leads to a bias. The R^2 declines rapidly with σ_η for regressions where Z_t^η contains only yields, and much more slowly if it includes yields and τ_t. The bias is the largest if Z_t^η contains three yields. It is somewhat smaller with six yields and yet smaller if we use three PCs. By constructing the PCs we diversify away part of the measurement error, however the bias still remains visibly larger than for a regression that includes yields jointly with τ_t. For instance, at $\sigma_\eta = 5$ basis points, the R^2 obtained with three PCs is 0.86, and the R^2 obtained with yields-plus-τ_t is 0.998. As such, in the yields-only regression, the bias in the coefficients resulting from measurement error implies that the estimated price-of-risk dynamics are significantly contaminated by the EH term; the EH term itself is uncorrelated with the true risk premium in the model.

It is more realistic to assume that an econometrician does not observe the true τ_t but uses a noisy proxy. At which size of the measurement error in τ_t^ϵ, σ_ϵ, would yields-plus-τ_t approach deliver a similar bias as the yields-only approach? In Panel B of Figure 5, we vary σ_ϵ between 0 and 50 basis points, keeping the measurement error in yields fixed at $\sigma_\eta = 9$ basis points. The figure shows that the bias in the yields-plus-τ_t regression deteriorates to the level of the three-PCs regression only at $\sigma_\epsilon = 49$ basis points.

We are not the first to suggest that measurement error can obscure the risk premium variation in yields (Cochrane and Piazzesi 2005; Duffee 2011). Cochrane and Piazzesi (2005) notice that one can improve on their predictive regressions by using additional lags of forward rates to mitigate the measurement error. We show that a small amount of noise in yields can explain why including more yields or forward rates as regressors, common in the literature, improves the return predictability, even if the true yield curve is driven by a small number of factors. Our results also illustrate that using a proxy for trend inflation, even if an imperfect one, is an effective strategy for uncovering the variation in risk premiums when measurement error is present. Intuitively, the measurement error affects most significantly our ability to recover the least persistent factor in yields (i.e., the price-of-risk factor). Providing additional information about trend inflation helps to disentangle the EH component in yields from the risk premium.

5. Cycles and CP Factor In and Out of Sample

Our predictability results naturally lead to a question about the link between the cycle factor and the single return forecasting factor of Cochrane and Piazzesi (2005).

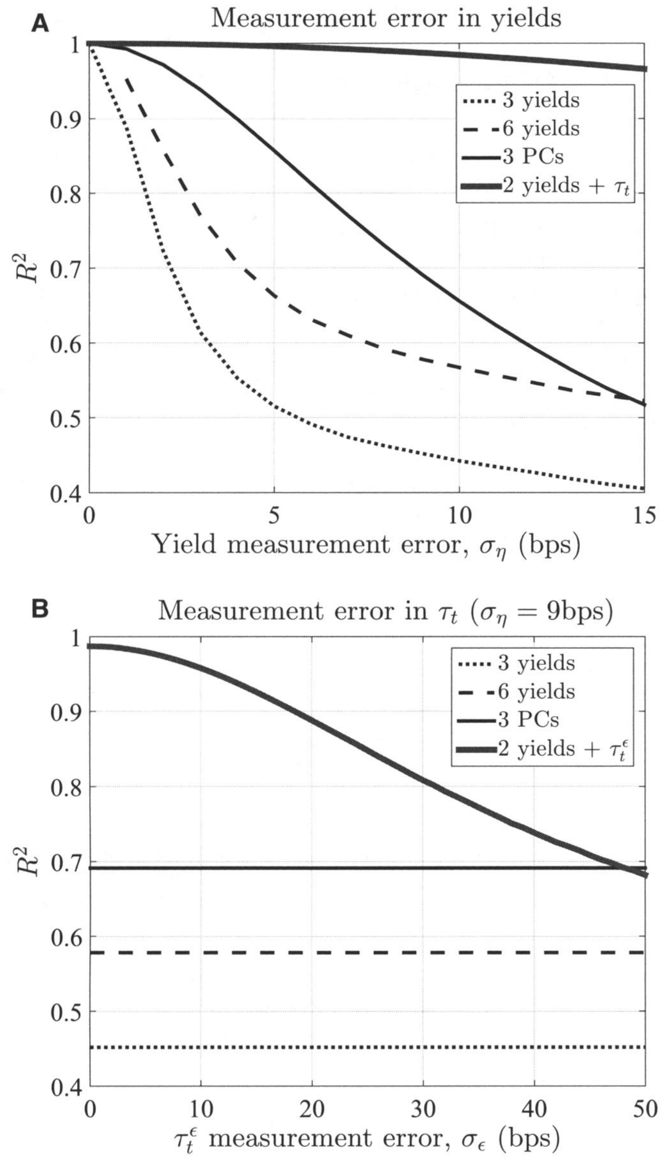

Figure 5
Bias resulting from measurement error
The figure presents the bias in recovering the price-of-risk factor x_t from the cross section of yields that arises because of the measurement error in yields and/or in trend inflation. Panel A plots the R^2 from Regression (34) (i.e., projecting x_t on Z_t^η). The legend displays the different specifications of Z_t^η. τ_t in regressions in Panel A is measured without an error. Panel B shows the R^2 from the same projection but when τ_t is measured with an iid normal noise (i.e., $\tau_t^\epsilon = \tau_t + \epsilon_t$). The simulation is based on the affine model from Section 1 using the parameters reported in Table 7. We simulate 470 monthly observations, with 10,000 repetition from the model. For each specification, the figure reports the average R^2's across simulations.

5.1 The Cochrane–Piazzesi factor

Cochrane and Piazzesi (2005) construct a return forecasting factor as the fitted value from the following regression:

$$\overline{rx}_{t+1}=\gamma_0+\sum_{i=1}^{m}\gamma_i f_t^{(i)}+\varepsilon_{t+1}=\gamma_0+\gamma'\mathbf{f}_t+\varepsilon_{t+1},\quad \bar{R}^2=0.24. \tag{35}$$

On the right-hand side, we use maturities of forward rates of 1, 2, 5, 7, 10 and 20 years. The explanatory power of the regressions is the same as in the yields-only specification in Equation (22), and using other maturities or more forward rates does not change the fit. We denote the Cochrane-Piazzesi (CP) factor as $CP_t=\hat{\gamma}'\mathbf{f}_t$. To investigate the link between our results and the CP regression let us express an n-year yield in terms of τ_t^{CPI} and the orthogonal residual $c_t^{(n)}$ (i.e., using Equation (21)):

$$y_t^{(n)}=\widehat{a}_n+\widehat{b}_n^{\tau}\tau_t^{CPI}+c_t^{(n)}. \tag{36}$$

Given that $f_t^{(n)}=-(n-1)y_t^{(n-1)}+ny_t^{(n)}$, any forward rate can be decomposed in an analogous way. Clearly, a projection of an n-year yield $y_t^{(n)}$ onto τ_t^{CPI} and $c_t^{(n)}$ gives an R^2 equal to one, and in this sense Equation (36) can be thought of as a decomposition. It is informative to rewrite the CP factor $\hat{\gamma}'\mathbf{f}_t$ in terms of τ_t^{CPI} and the cycles:

$$\hat{\gamma}'\mathbf{f}_t=\widetilde{\gamma}_0+\tau_t^{CPI}\Big(\sum_{i=1}^{m}\overline{\gamma}_i\Big)+\sum_{i=1}^{m}\widetilde{\gamma}_i c_t^{(i)}=\widetilde{\gamma}_0+\overline{\gamma}'\mathbf{1}\tau_t^{CPI}+\widetilde{\gamma}'\mathbf{c}_t, \tag{37}$$

where vectors $\overline{\gamma}$ and $\widetilde{\gamma}$ are functions of the estimated $\hat{\gamma}$'s in Equation (35) and $\widehat{b}_n^{\tau}$'s in Equation (36),[16] $\mathbf{1}$ is an m-dimensional vector of ones, and $\mathbf{c}_t=\left(c_t^{(1)},\ldots,c_t^{(m)}\right)'$. By construction, the correlation between $\hat{\gamma}'\mathbf{f}_t$ in Equation (35) and $(\overline{\gamma}'\mathbf{1}\tau_t^{CPI}+\widetilde{\gamma}'\mathbf{c}_t)$ in Equation (37) equals one.

From Section 4.1, one can expect that the CP factor captures mainly the variation in yields independent of the trend inflation. This can be tested by allowing the excess return to load with separate coefficients on each term in Equation (37):

$$\overline{rx}_{t+1}=a_0+\underbrace{a_1}_{\substack{-0.68\\(-0.07)}}\underbrace{(\overline{\gamma}'\mathbf{1}\tau_t^{CPI})}_{\text{1st regressor}}+\underbrace{a_2}_{\substack{1.1\\(3.36)}}\underbrace{(\widetilde{\gamma}'\mathbf{c}_t)}_{\text{2nd regressor}}+\varepsilon_{t+1},\quad \bar{R}^2=0.27. \tag{38}$$

The entire explanatory power of Regression (38) comes from the second term. We note that Regression (38) is not equivalent to the unconstrained yields-plus-τ_t^{CPI} Regression (23) because the $\bar{\gamma}$ and $\widetilde{\gamma}$ coefficients are determined by the

16 The coefficients are $\overline{\gamma}_k=\hat{\gamma}_k\left[-(k-1)\widehat{b}_{k-1}^{\tau}+k\widehat{b}_k^{\tau}\right]$ and $\widetilde{\gamma}_k=\begin{cases} k(\hat{\gamma}_k-\hat{\gamma}_{k+1}) & \text{for } 1\le k<m \\ k\hat{\gamma}_k & \text{for } k=m. \end{cases}$

Table 8
Bivariate predictive regressions with the CP and the cycle factor

A. CMT zero-coupon yields

	$rx^{(2)}$	$rx^{(5)}$	$rx^{(7)}$	$rx^{(10)}$	$rx^{(15)}$	$rx^{(20)}$
CP_t	−0.01	−0.05	−0.02	−0.03	0.00	0.07
	(−0.88)	(−0.41)	(−0.23)	(−0.18)	(0.03)	(0.36)
$\widehat{cf}_t$	0.63	0.71	0.72	0.75	0.74	0.67
	(3.55)	(3.84)	(3.93)	(4.05)	(3.99)	(3.63)
$\bar{R}^2$ (CP + $\widehat{cf}$)	0.38	0.46	0.49	0.53	0.54	0.51
R^2 from univariate regressions:						
R^2 (CP)	0.17	0.19	0.22	0.22	0.25	0.27
R^2 ($\widehat{cf}$)	0.38	0.46	0.49	0.53	0.54	0.51

B. Fama–Bliss zero-coupon yields

	$rx^{(2)}$	$rx^{(3)}$	$rx^{(4)}$	$rx^{(5)}$
CP_t	0.07	0.09	0.13	0.08
	(−0.28)	(0.09)	(0.37)	(0.22)
$\widehat{cf}_t$	0.56	0.56	0.56	0.59
	(4.02)	(4.21)	(4.36)	(4.51)
$\bar{R}^2$ (CP + $\widehat{cf}$)	0.36	0.39	0.41	0.41
R^2 from univariate regressions:				
R^2 (CP)	0.17	0.19	0.22	0.20
R^2 ($\widehat{cf}$)	0.36	0.38	0.41	0.41

Table 8 reports the results from bivariate predictive regressions of bond excess returns with the CP factor and the $\widehat{cf}$ factor as regressors: $rx_{t+1}^{(n)} = \alpha + \beta_1 CP_t + \beta_2 \widehat{cf}_t + \varepsilon_{t+1}$. The last two rows of each panel provide the predictive R^2 from univariate regressions using either the CP factor or the $\widehat{cf}$ factor as the regressor. Panel A uses CMT-based zero-coupon yields (forwards) with maturities of 1, 2, 5, 7, 10, and 20 years, to construct $\widehat{cf}_t$ and $CP_t = \hat{\gamma}'\mathbf{f}_t$ factors, as in Equations (27) and (35), respectively. Analogously, Panel B uses Fama-Bliss zero-coupon yields with maturities from 1 through 5 years to construct the forecasting factors. Reverse regression delta method t-statistics are in parentheses. All variables are standardized.

$\hat{\gamma}$'s estimated in Equation (35). The intuition from the affine model about the role of measurement error in the yields-only regressions suggests that the CP factor is noised up by the variation in the EH component of yields. Indeed, the correlation of $\hat{\gamma}'\mathbf{f}_t$ with the two EH terms in our decomposition, τ_t^{CPI} and $c_t^{(1)}$, is 0.20 and −0.56, respectively, and its correlation with $\widehat{cf}_t$ is 0.67.

Table 8 reports bivariate predictive regressions in which we include both the cycle factor $\widehat{cf}_t$ and the CP factor as regressors. There is evidence that CP regressions can be sensitive to the measurement error that arises from different ways of constructing the zero-coupon yield curve (Cochrane and Piazzesi 2008). Therefore, we provide estimates for the CMT, as well as for the Fama–Bliss data sets. The conclusions are the same when the GSW yields are used and are not reported to save space. The main observation from the table is that the CP factor becomes economically and statistically insignificant and has no incremental explanatory power beyond $\widehat{cf}_t$. This conclusion holds for all bond maturities and in both data sets, and is consistent with these two return-forecasting variables capturing a common source of variation in expected bond returns, but the CP factor being more noisy.

5.2 Out-of-sample predictability

We conduct an out-of-sample experiment to assess the stability of the forecasts over time. Suppose that an investor forms inflation expectations using the DMA τ_t^{CPI} as a rule of thumb. In doing so, he exploits inflation information that is available up to time t and updates the estimates of τ_t^{CPI} with the incoming data and the gain parameter of $v = 0.987$. Note that we endow the investor with the learning parameter v; therefore, the exercise is quasi out of sample. We drop this assumption in Section 6.2.

We start the out-of-sample exercise at three dates, January 1978, January 1985, and January 1995, all ending in December 2011, and consider three sets of forecasting variables: cycles, forwards, and forward-spot spreads. Because we forecast annual returns, for each sample, we obtain the initial parameter estimates based on the periods from November 1971 until January of 1977, 1984, and 1994, respectively. Using information up to, say t_0, we predict bond returns at t_0+12 months and expand the sample month by month.[17] Our out-of-sample tests involve the following three statistics:

The encompassing test (ENC-NEW) of Clark and McCracken (2001)—the null hypothesis is that the forward rate regressions encompass all predictability in bond excess returns, which cannot be further improved by conditioning on trend inflation.

The ratio of mean squared errors (MSEs) implied by the forward rate versus cycle regressions—a number less than one indicates that cycles generate a lower prediction error.

The out-of-sample R^2 of Campbell and Thompson (2008), R^2_{oos}, which compares the forecasting performance of a given predictor toward a "naive" forecast obtained with the historical average return—a positive value indicates that the predictive model has a lower mean-squared prediction error than the "naive" forecast.

For forward rate regressions, we use rates with maturities of 1, 2, 5, 7, 10, and 20 years. For cycle regressions, we use two specifications differing in the number of regressors: (i) six cycles with the same maturities as forward rates and (ii) two cycles, $c_t^{(1)}$ and $\bar{c}_t$, as used to construct $\widehat{cf}_t$ in Equation (27). Although (i) and (ii) produce essentially identical in-sample results, comparing them out of sample helps assess the effect of overfitting on the stability of the forecasts. For comparison with the forward rate regressions, in the ENC test we employ the six-cycles specification. Finally, for forward-spot

17 With information up to time t_0, we obtain cycles as in Equation (21), and run regressions to predict excess returns realized in t_0 with data up to $t_0 - 12$ months. At the estimated parameters, we then predict excess returns 12 months ahead (i.e., realized at t_0+12 months). We extend the sample month by month and repeat these steps until we reach the maximum sample length.

spread regressions, we predict excess returns on the n-year bond with the corresponding forward-spot spread, $f_t^{(n)} - y_t^{(1)}$.

Table 9 reports the out-of-sample tests. The ENC-NEW test rejects the null hypothesis for all maturities at the 95% confidence level: using cycles significantly improves the forecasts over the linear combination of forward rates. For all maturities, forward-rate regressions have higher MSEs and deliver negative out-of-sample R^2_{oos}. This is consistent with the recent evidence of Thornton and Valente (2012), who argue that using multiple forward rates to predict bond excess returns does not generate systematic economic value to investors out of sample and can be outperformed with a simple forward-spot spread. However, our two-cycles specification outperforms the forward-spot spread. A comparison of the R^2_{oos} between the two- and six-cycle models suggests that overfitting is a concern in predictive regressions with several correlated regressors. A more parsimonious specification gives results that are more consistent across subsamples and maturities. With one exception, R^2_{oos} obtained with two cycles are positive, whereas the six-cycles version deteriorates in the last subperiod along with other predictors.

The negative out-of-sample results using multiple forward rates do not mean that the in-sample evidence using the CP factor is spurious (see also Inoue and Kilian (2004)). Rather, although the CP factor captures an important element of the risk premium, its coefficient estimates are noisy, creating a wedge between in- and out-of-sample forecasts. From this perspective, one useful feature of our approach is that it involves few precisely estimated parameters.

In the Online Appendix, we show that the out-of-sample conclusions with cycles are not significantly changed when varying the learning parameter v in Equation (19) between 0.975 (fast updating) and 0.995 (very slow updating), which is the range of values used in the literature.

6. Robustness

6.1 Alternative measures of inflation expectations

In this section, we discuss alternative proxies for trend inflation based on survey inflation expectations, current realized inflation, and different ways of smoothing past realized inflation. In each case, we reconstruct the cycle factor by replacing τ_t^{CPI} with an alternative measure and reestimate our return predictive regressions for maturities of 2, 5 and 10 years. The conclusions for other maturities remain unchanged and therefore we do not report those for brevity. The results are summarized in Table 10.

In Panels A and B of Table 10, we consider survey expectations of inflation from several sources (Blue Chip, SPF, and Livingston surveys). The surveys are available at different frequencies, different forecast horizons (long-horizon forecasts in Panel A and 1-year forecasts in Panel B), and over varying sample periods. Details regarding the data are provided in the Online Appendix. We

Table 9
Out-of-sample tests

Test	$rx^{(2)}$	$rx^{(5)}$	$rx^{(7)}$	$rx^{(10)}$	$rx^{(15)}$	$rx^{(20)}$
A. Out-of-sample period: 1978–2011						
(1) ENC-NEW	139.69	146.19	157.23	176.97	176.40	157.61
(2) Bootstrap 95% CV	75.99	61.93	60.60	57.37	57.62	57.55
(3) MSE(6 cyc)/MSE(6 fwd)	0.70	0.63	0.60	0.56	0.56	0.62
(4) MSE(2 cyc)/MSE(6 fwd)	0.66	0.57	0.55	0.50	0.50	0.63
(5) R^2_{OOS}(6 cyc)	0.10	0.19	0.26	0.30	0.31	0.31
(6) R^2_{OOS}(2 cyc)	0.16	0.27	0.33	0.38	0.38	0.30
(7) R^2_{OOS}(6 fwd)	−0.29	−0.28	−0.23	−0.25	−0.24	−0.12
(8) R^2_{OOS}(fwd-spot spread)	0.10	0.09	0.07	0.08	0.09	0.09
B. Out-of-sample period: 1985–2011						
(1) ENC-NEW	116.49	115.58	127.23	145.34	146.33	134.47
(2) Bootstrap 95% CV	48.52	39.18	39.04	39.81	41.76	44.06
(3) MSE(6 cyc)/MSE(6 fwd)	0.69	0.64	0.62	0.60	0.62	0.67
(4) MSE(2 cyc)/MSE(6 fwd)	0.59	0.52	0.52	0.50	0.52	0.59
(5) R^2_{OOS}(6 cyc)	−0.08	0.16	0.25	0.30	0.31	0.31
(6) R^2_{OOS}(2 cyc)	0.08	0.31	0.37	0.42	0.42	0.39
(7) R^2_{OOS}(6 fwd)	−0.55	−0.31	−0.20	−0.17	−0.11	−0.04
(8) R^2_{OOS}(fwd-spot spread)	0.07	0.07	0.05	0.08	0.09	0.10
C. Out-of-sample period: 1995–2011						
(1) ENC-NEW	61.65	57.88	66.75	81.24	86.43	81.53
(2) Bootstrap 95% CV	22.28	23.11	25.34	24.53	27.28	36.22
(3) MSE(6 cyc)/MSE(6 fwd)	0.67	0.64	0.59	0.52	0.50	0.54
(4) MSE(2 cyc)/MSE(6 fwd)	0.46	0.46	0.42	0.41	0.38	0.33
(5) R^2_{OOS}(6 cyc)	−0.58	−0.26	−0.16	−0.03	0.01	−0.12
(6) R^2_{OOS}(2 cyc)	−0.09	0.10	0.17	0.20	0.26	0.32
(7) R^2_{OOS}(6 fwd)	−1.37	−0.97	−0.97	−0.97	−0.97	−1.07
(8) R^2_{OOS}(fwd-spot spread)	−0.19	−0.06	−0.09	−0.06	−0.06	−0.06

Table 9 reports the results of out-of-sample tests. Row (1) in each panel contains the ENC-NEW test. The null hypothesis is that the predictive regression with forward rates encompasses all predictability in bond excess returns. The null is tested against the alternative that cycles improve the predictability achieved by forward rates. For forward rates and cycles we use maturities of 1, 2, 5, 7, 10, and 20 years. Row (2) reports bootstrapped critical values (CV) for the ENC-NEW statistic at the 95% confidence level (see the Online Appendix for implementation details). Rows (3) and (4) show the ratio of mean squared errors for the cycles and forward-rate models, Row (3) with six cycles and Row (4) with two cycles, $\bar{c}_t, c_t^{(1)}$. Rows (5)–(8) report the out-of-sample R^2, R^2_{OOS}, for two and six cycles, forwards rates and the forward-spot spread, respectively. For forward-rate and six-cycles specifications we use the same six maturities, for two cycles we use $c_t^{(1)}$ and $\bar{c}_t$ as in the construction of forecasting factor $\widehat{cf}_t$. The forward-spot spread used for predicting the bond return with maturity n is constructed as $f_t^{(n)} - y_t^{(1)}$.

retain the frequency at which survey forecasts are reported to avoid look-ahead bias resulting from interpolation. We also construct the cycle factor with τ_t^{CPI} corresponding to the sample period and frequency of the survey, to allow comparison with previous sections, and report the corresponding $\bar{R}^2$ (row "$\bar{R}^2(\widehat{cf})$"). In rows labeled "pval Wald ($\widetilde{\tau}$)" we report the p-values for a test that the coefficient on the alternative trend-inflation measure, $\widetilde{\tau}_t$, is equal to zero. The test is based on regression of excess return on $\bar{y}_t$, $y_t^{(1)}$ and $\widetilde{\tau}_t$ analogous to specification given in Equation (24).

Overall, survey-based measures of expected inflation bring in new information to the regressions in addition to the information contained in yields. We reject the null that the loading on survey expected inflation is zero at the 1%

Table 10
Different measures of trend inflation

	$rx^{(2)}$	$rx^{(5)}$	$rx^{(10)}$	$rx^{(2)}$	$rx^{(5)}$	$rx^{(10)}$
A. 10-year-ahead expected inflation from surveys						
	A1. CPI infl. Livingston + BCEI 1979-2011, semiannual (65* obs.)			A2. CPI infl. BCEI (5-10Y) 1979-2011, semiannual (62* obs.)		
(1) $\widetilde{cf}_t$	0.60	0.60	0.64	0.63	0.61	0.66
	(2.39)	(3.05)	(3.62)	(2.85)	(3.35)	(3.66)
(2) $\bar{R}^2$	0.35	0.35	0.40	0.38	0.36	0.43
(3) pval Wald ($\widetilde{\tau}$)	0.00	0.00	0.00	0.00	0.00	0.00
(4) $\bar{R}^2(\widehat{cf})$	0.23	0.36	0.45	0.24	0.37	0.47
B. 1-year ahead expected inflation from surveys						
	B1. CPI infl., BCFF 1984-2011, monthly (318 obs.)			B2. GDP deflator, SPF 1971-2011, quarterly (156 obs.)		
(1) $\widetilde{cf}_t$	0.47	0.55	0.62	0.52	0.49	0.54
	(1.93)	(2.51)	(2.60)	(2.68)	(3.04)	(3.30)
(2) $\bar{R}^2$	0.21	0.31	0.38	0.27	0.23	0.28
(3) pval Wald ($\widetilde{\tau}$)	0.02	0.01	0.01	0.00	0.00	0.00
(4) $\bar{R}^2(\widehat{cf})$	0.24	0.39	0.50	0.35	0.44	0.51
C. Realized inflation measures, 1971–2011 (monthly, 470 obs.)						
	C1. Current yoy core CPI infl.			C2. Current yoy all-items CPI infl.		
(1) $\widetilde{cf}_t$	0.43	0.42	0.48	0.38	0.37	0.41
	(1.80)	(2.25)	(2.54)	(1.60)	(2.01)	(2.29)
(2) $\bar{R}^2$	0.18	0.18	0.23	0.15	0.13	0.17
(3) pval Wald ($\widetilde{\tau}$)	0.13	0.09	0.05	0.10	0.08	0.05
	C3. MA_{60} of core CPI infl.			C4. DMA of all-items CPI infl.		
(1) $\widetilde{cf}_t$	0.63	0.63	0.68	0.57	0.60	0.66
	(3.56)	(4.32)	(4.81)	(3.08)	(3.92)	(4.53)
(2) $\bar{R}^2$	0.39	0.40	0.47	0.32	0.36	0.43
(3) pval Wald ($\widetilde{\tau}$)	0.00	0.00	0.00	0.00	0.00	0.00

Table 10 summarizes predictive regression using the cycle factor constructed with different measures of trend inflation. We use notation $\widetilde{cf}_t$ to indicate that the cycle factor is constructed using an alternative proxy to τ_t^{CPI}, distinguishing it from the $\widehat{cf}_t$ used above. The regression coefficients in Row (1) are standardized. Row (3) "pval Wald ($\widetilde{\tau}$)" shows p-values for a test that the coefficient on the particular trend-inflation measure $\widetilde{\tau}_t$ is zero in a regression specified as in Equation (24), where excess return is projected on $y_t^{(1)}$, $\bar{y}_t$ and $\widetilde{\tau}_t$. All t-statistics and Wald tests are based on the reverse regression delta method. Because survey samples have different lengths and frequencies, in Row (4) of Panels A and B, we report the R^2 from cycle factor $\widehat{cf}_t$ constructed with τ_t^{CPI} and sampled over the same period and frequency as a given inflation survey. Results in Panel A are based on long-term surveys (panel A1 uses data from two surveys as merged by the Philadelphia Fed). Results in Panel B are for 1-year ahead surveys, and in Panel C are for the realized inflation measures where yoy denotes year-over-year inflation. * The different numbers of observations in Panels A1 and A2 are owing to the way Philadelphia Fed combines the two sources (in Panel A1 we gain two observations), and because of a missing BCEI survey in October 1998 (in Panel A2 we lose one observation).

level or better in most cases. The predictive power of the cycle factor obtained with long-horizon surveys (Panel A) lines up closely with the results based on τ_t^{CPI}. Even though short-horizon inflation forecasts are also highly significant (Panel B), they deliver somewhat weaker predictability relative to long-horizon inflation forecasts. To understand the differences across survey horizons, we note that short-horizon surveys are typically less persistent and more volatile compared with long-horizon surveys or τ_t^{CPI}, whereas evidence suggests that transitory fluctuations in inflation in the last two decades do not pass through onto interest rates (Stock and Watson 2011; Ajello, Benzoni, and Chyruk 2012).

Contrary to the survey-based results, conditioning on realized inflation (core or all-items) gives essentially the same amount of return predictability as the yields-only regressions do (Panel C1 and C2 of Table 10). The loadings on current year-on-year inflation are marginally significant with p-values exceeding 5%.

In Panel C, we also study two alterations to the baseline τ_t^{CPI}: a simple 60-month MA of core CPI inflation (Panel C3), and a DMA of all-items CPI inflation (Panel C4). In both cases, we find strong evidence of predictability and highly significant loading on the trend inflation (p-values below 0.1%). The DMA applied to all-items inflation delivers a 9% lower R^2 on average across maturities than when constructed with core inflation, but the R^2 is still robustly higher than it is in the yields-only regressions. As the source of the discrepancy, all-items and core inflation diverged visibly in the 2000s, owing to the elevated volatility of energy prices. Towards the end of our sample, τ_t^{CPI} from core inflation remained stable around 2%–2.5%, and surveys recorded essentially flat long-run inflation expectations, despite survey participants predicting all-items inflation. In contrast, the DMA of all-items CPI inflation increased above 3%. Both survey forecasts and the evidence of a limited pass-through of energy prices onto the yield curve in that period suggest that the private sector perceived the volatility of energy prices as a transitory phenomenon that does not alter the underlying inflation trend. Even though these arguments may speak in favor of using core CPI inflation to obtain τ_t^{CPI}, the main conclusions from the preceding sections remain valid with all-items CPI inflation, as well.

The main conclusion from Table 10 is that our approach to estimating the risk-premium variation holds for different measures of trend inflation, especially those capturing the low-frequency dynamics of inflation expectations.

6.2 Long historical samples and short-horizon surveys

The analysis up to this point has relied on a sample beginning in November 1971, when bonds with maturities of 10 years and above become available, and which contain substantial information about the risk premium. For this reason, several studies have focussed on the post-1970 period (e.g., Cochrane and Piazzesi 2008; Duffee 2010; Le and Singleton 2013). Additionally, our τ_t^{CPI} proxy can only be constructed starting in 1967 because of the availability of the core CPI data. It is nevertheless informative to consider longer samples, as well. In this section, we report results going back to the beginning of the Fama–Bliss data set in 1952, spanning the entire period post Fed-Treasury Accord. As a measure of trend inflation, we use the 12-month-ahead median forecast of all-items CPI inflation (nonseasonally adjusted) from the Livingston survey (denoted as Liv_t), which is the only inflation survey compiled over the entire sample. Throughout, we do not use any interpolated data, and we run regressions at the semiannual frequency at which the Livingston survey is conducted (in June and December each year).

Table 11 reproduces the key results from the previous sections in this long sample, and confirms our main conclusions. Panel A reports regressions of $\overline{rx}_{t+1}$ on $y_t^{(1)}, \bar{y}$ and Liv_t, analogous to Regression (24), with and without coefficient restrictions. All three variables are highly statistically significant and the unconstrained regressions is selected as the preferred model by the BIC (Column (1)). Restricting the coefficient on $\bar{y}_t$ to zero removes essentially all the predictive power of the regression (Column (2)). This implies that expected inflation and the 1-year yield capture the EH term that is nearly orthogonal to the risk premium, in line with our previous conclusions.[18]

Panels B and C of Table 11 compare the predictability of individual bond excess returns using cycles and yields, respectively. The cycle factor is a significant predictor of bond excess returns at all maturities (Rows 1–2, Panel B). It explains effectively the same amount of excess return variation as the less parsimonious yields-plus-Liv_t specification (Row 3). For each maturity, we strongly reject the null hypothesis of no predictability (Row 4) and, more importantly, the null that the loading on expected inflation, Liv_t, is zero (Row 5). In sample, the cycles' approach explains on average 10% more of return variation than the yields-only (CP) approach. The differences become larger out of sample, where cycles deliver positive R^2_{oos} ranging from 16% for the 2-year bond to 9% for the 5-year bond, whereas yields-only regressions deliver negative R^2_{oos} ranging from -15% to -9% for the corresponding maturities (Row 6). Finally, including the past smoothed Tbill rate, in analogy to Regression (26), does not contain additional information about excess bond returns over and above expected inflation (Row 7).

In Panel D of Table 11, we estimate bivariate predictive regressions of returns confirming that $\widehat{cf}_t$ drives out the predictive content of the CP factor. The explanatory power of the regression remains unchanged relative to the univariate case with $\widehat{cf}_t$ only.

Compared with the previous sections, the tests summarized here are conservative in several respects. We rely on short-horizon survey forecasts of all-items inflation that are likely to overstate the volatility of trend inflation that is reflected in the yield curve. We also use yields with a maximum maturity of 5 years. Even though these results confirm all of our main conclusions, the predictability in terms of $\bar{R}^2$ is more conservative. Yet, it is clearly above the 95th quantile of $\bar{R}^2$ obtained under the null of EH. We interpret it as a benchmark for a plausible degree of return predictability in the Treasury bond market.

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[18] The marginal significance of the regressors in Column (2) is driven by the inclusion of excess returns with short maturities (the 2-year bond) in the $\overline{rx}$. For the maximum maturity in this sample period (i.e., 5 years), we fail to find statistically significant predictability of excess returns with Livingston expected inflation and 1-year yield. The results for individual maturities are not reported for brevity.

Table 11
Long sample with Livingston inflation survey, 1952–2011

A. Predictive regressions: $\overline{rx}_{t+1} = d_0 + d_1 y_t^{(1)} + d_2 \bar{y}_t + d_3 \text{Liv}_t + \varepsilon_{t+1}$

	(1)	(2)	(3)	(4)
$y_t^{(1)}$	−0.65	0.15	–	−0.66
	(−2.83)	(2.26)	–	(−3.01)
$\bar{y}_t$	0.93	–	0.27	0.74
	(3.70)	–	(3.09)	(3.12)
Liv_t	−0.26	−0.17	−0.27	–
	(−3.12)	(−2.36)	(−3.14)	–
$\bar{R}^2$	0.28	0.05	0.15	0.15
Wald test	15.37	5.83	10.39	9.73
pval	0.00	0.05	0.01	0.01
Rel.prob. (BIC)	1.00	0.00	0.00	0.00

B. Cycle factor, $\widehat{cf}$	$rx^{(2)}$	$rx^{(3)}$	$rx^{(4)}$	$rx^{(5)}$	C. Cochrane–Piazzesi (CP) factor	$rx^{(2)}$	$rx^{(3)}$	$rx^{(4)}$	$rx^{(5)}$
(1) $\widehat{cf}_t$	0.54	0.53	0.54	0.54	(1) CP_t	0.42	0.44	0.46	0.45
	(3.36)	(3.71)	(3.99)	(4.20)		(2.64)	(3.55)	(3.84)	(3.92)
(2) $\bar{R}^2(\widehat{cf})$	0.29	0.28	0.29	0.29	(2) $\bar{R}^2(CP)$	0.17	0.19	0.20	0.19
(3) $\bar{R}^2$(5yld+Liv)	0.27	0.29	0.29	0.29	(3) $\bar{R}^2$(5yld)	0.15	0.16	0.17	0.17
(4) Wald, $\chi^2(6)$	13.59	20.48	20.56	24.17	(4) Wald, $\chi^2(5)$	7.72	15.31	16.35	19.16
pval	0.03	0.00	0.00	0.00	pval	0.17	0.01	0.01	0.00
(5) Wald (Liv=0)	7.38	8.10	8.23	9.96		–	–	–	–
pval	0.01	0.00	0.00	0.00		–	–	–	–
(6) R^2_{oos}(2 cyc)	0.16	0.09	0.11	0.09	(6) R^2_{oos} (5 fwd)	−0.15	−0.12	−0.10	−0.09
(7) pval(τ^{y3m})	0.24	0.20	0.16	0.13		–	–	–	–
$\bar{R}^2$(Liv+τ^{y3m})	0.28	0.27	0.28	0.29		–	–	–	–

D. Bivariate regressions with CP and $\widehat{cf}$

	$rx^{(2)}$	$rx^{(3)}$	$rx^{(4)}$	$rx^{(5)}$
$\widehat{cf}_t$	0.52	0.46	0.46	0.49
	(2.77)	(2.51)	(2.62)	(2.84)
CP_t	0.04	0.10	0.11	0.08
	(−0.90)	(0.11)	(0.14)	(0.16)
$\bar{R}^2$	0.28	0.28	0.29	0.29

Table 11 reports results for predictive regressions over the 1952–2011 sample. To construct cycles, we use 1-year-ahead inflation forecasts from the Livingston survey ("Liv_t"). Yields are from the Fama–Bliss data set, with maturities available from 1 though 5 years. The data are semiannual becuase of the frequency of the Livingston survey (120 observations). Panel A reports the regressions analogous to Equation (24), where in Columns (2)–(4) some coefficients have been restricted to zero. In Panels B and C the rows are: (1) standardized regression coefficient on the single forecasting factor, (2) in-sample $\bar{R}^2$ for the single factor, (3) $\bar{R}^2$ for unconstrained regressions on five yields with and expected inflation ("5 yld+Liv" in Panel B) and only five yields ("5 yld" in Panel C), (4) Wald statistic testing the null of no predictability, (5) Wald statistic testing zero coefficient on expected inflation (only Panel B), (6) out-of-sample R^2, R^2_{oos}, according to specification with $c_t^{(1)}, \bar{c}$ in Panel B and five forwards in Panel C. The out-of-sample exercise uses a burn-in period of 20 years (i.e., 40 semiannual observations), (7) statistics from regressions with two trends (inflation and smoothed past short rate) as in Equation (26), where τ^{y3m} is the DMA of the 3-month T-bill rate constructed as in Equation (19) on monthly data and is sampled semiannually. We only report the p-value of the corresponding coefficient, pval(τ^{y3m}), and $\bar{R}^2$ from the regression. Panel D estimates a bivariate regression with CP and $\widehat{cf}_t$ factors included jointly, and coefficients are standardized. The t-statistics in parentheses and Wald tests are obtained from the reverse regression delta method.

6.3 Risk premiums implied by interest rate surveys

Finally, we use direct forecasts of interest rates from surveys (SPF and Blue Chip) to compare the survey-implied expected returns with our measure of the risk premium—the cycle factor. Interest rate surveys are available from the 1980s (the start date depends on the source), and they differ in terms of maturities of interest rates being forecast, horizons, and the frequency at which they are compiled. For brevity, the details on the survey data, and how we construct the risk/term premium using them, are reported in the Online Appendix. Because surveys reflect real-time expectations, for sake of comparison, we construct the cycle factor also in real time, $\widehat{cf}_t^{RT}$ (i.e., reestimating cycles and Equation (27) recursively month by month). We obtain $\widehat{cf}_t^{RT}$ with two zero-coupon yield data sets: CMT and Fama-Bliss. The correlation among various survey-based risk premium measures ranges from 0.36 for 10-year maturity to 0.75 for 2-year maturity. The correlation of survey-based measures with the $\widehat{cf}_t^{RT}$ variable ranges from 0.34 for 10-year maturity to 0.76 for 2-year maturity (see the Online Appendix for details). The consistently positive correlations between the statistical and the survey-based risk premiums are in contrast with the findings in the equity literature. In particular, Greenwood and Shleifer (2013) show that, even though expected equity returns inferred from various investor surveys are positively correlated with each other, they are *negatively* correlated with the standard model-based measures of expected returns such as the dividend-price ratio.

In Table 12, we compare the properties of expected returns from the Blue Chip Financial Forecasts (BCFF) survey with expected returns implied by the cycle factor.[19] Expected returns from the survey are obtained as:

$$E_t^s(rx_{t+1}^{(n)}) = -(n-1)E_t^s(y_{t+1}^{(n-1)}) + ny_t^{(n)} - y_t^{(1)}, \tag{39}$$

where $E_t^s(\cdot)$ is the survey-based expectation. We report results for $n=2,5,10$. Panel A of Table 12 confirms the positive correlation of the cycle factor with the expected return from the survey, which is highest at the 2-year maturity. Panel B displays the volatility of expected returns fitted with $\widehat{cf}_t^{RT}$ and compares it to the volatility of survey-based expected returns. The row labeled "ratio" gives the ratio of the respective standard deviations. Expected excess returns implied by $\widehat{cf}_t^{RT}$ have a lower volatility than the survey has at the 2-year maturity and a higher volatility than the survey has at the 10-year maturity, but overall they are of comparable magnitudes.

Our previous discussion shows that the risk premium is the least persistent component in yields and, relative to the EH term, it accounts for a smaller portion of the yield variance. Therefore, its estimation can be particularly sensitive to measurement error. The measurement error in survey-implied expected returns is likely to be nonnegligible because expected returns reflect small spreads between yields scaled by duration, as visible in Equation (39).

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[19] BCFF provides 1-year ahead forecasts of yields with maturities 3 and 6 months, and 1, 2, 5, 10, and 30 years. The data is available monthly from January 1988. Because forecasters predict CMT yields (i.e., coupon yields), we construct the expected zero-coupon yields with a yearly spacing of maturities by bootstrapping the survey expectations.

Table 12
Comparison of the cycle factor and survey-based expected returns

	CMT			FB		
	2Y	5Y	10Y	2Y	5Y	10Y
A. Correlation of survey expected returns with the cycle factor						
corr (BCFF, $\widehat{cf}^{RT}$)	0.59	0.54	0.40	0.73	0.59	0.49
pval	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
B. Standard deviations of expected returns						
st.dev. ($\widehat{cf}^{RT}$), % p.a.	0.55	2.17	4.46	0.52	1.85	3.86
ratio	0.90	1.27	1.36	0.86	1.09	1.18
C. $\bar{R}^2$ from predictive regressions of bond returns						
$\bar{R}^2$(BCFF)	0.05	0.05	0.03	—	—	—
$\bar{R}^2(\widehat{cf}^{RT})$	0.16	0.24	0.33	0.15	0.17	0.25

The table compares the survey-based expected bond excess returns with the cycle factor, $\widehat{cf}_t^{RT}$, constructed recursively in real time. Survey-based expected returns are obtained from the Blue Chip Financial Forecasts (BCFF), available monthly for the sample period from January 1988 through December 2010. The results are presented for the cycle factor constructed using two data sets: CMT and Fama-Bliss (FB). CMT contains maturities up to 20 years, whereas the longest maturity in FB is 5 years. Panel A reports the unconditional correlation between the survey-based risk premiums and the cycle factor. Panel B reports the standard deviation of expected returns from the survey, and expected returns fitted by the cycle factor regression. The row labeled "ratio" reports the ratio of the standard deviations of expected return fitted with the return forecasting factor to the expected return from the survey. A ratio below one means that $\widehat{cf}_t^{RT}$ underestimates the volatility of the expected return relative to the survey. Panel C reports the $\bar{R}^2$ from the predictive regression of realized returns. In both CMT and FB panels, we predict returns computed from the CMT zero-coupon yields to be able to consider the 10-year bond. Thus, CMT and FB panels differ by the dataset used to construct the $\widehat{cf}_t^{RT}$ factor, but predict the same excess returns. All results are for monthly data and sample period 1988–2010, corresponding to survey availability.

Panel C of Table 12 presents the $\bar{R}^2$ from the predictive regressions of realized excess returns on the expected excess returns from the BCFF survey. Indeed, the return predictability obtained using surveys is a magnitude lower than the one obtained with $\widehat{cf}_t^{RT}$ over the same sample period, suggesting that surveys provide a noisier measure of the bond risk premium than the cycle factor does.

7. Conclusions

We construct a measure of variation in expected returns on Treasury bonds, starting from the basic decomposition of yields into the EH term and the term premium. We separate two effects in yields across maturities: a common slow-moving part resulting from expected inflation, and an orthogonal, more transitory variation specific to each maturity, which we label as cycle. We show empirically that two elements—expected inflation and the short-maturity cycle—can be used to control for the expectations hypothesis term and that neither of them forecasts future returns. The short-maturity cycle covaries with measures of the *ex-ante* real rate. At longer maturities, the cycles embed both short-rate expectations and the risk premium, and their predictive power for bond returns increases with the maturity. Using this observation, we construct the return forecasting factor, which we call the cycle factor. The cycle factor forecasts bond excess returns across the entire maturity spectrum, both in and out of sample, subsuming other standard bond predictors used in the literature such as the term spread or the linear combination of forward rates.

We leave open the question of the economic determinants of the risk premium in Treasury bonds. The high level of predictability and the relatively rapid mean reversion in the risk premium that we identify make these results difficult to generate within standard consumption-based asset pricing models. Yet, our findings are consistent with the observation in the literature that a large share of variation in the Treasury risk premium is orthogonal to macro variables and standard measures of risk (e.g., Duffee 2013). Reconciling the empirical risk premium dynamics with macro aggregates and with the special features of the Treasuries such as safety and liquidity (Krishnamurthy and Vissing-Jorgensen 2012) or compensation for bearing duration risk (Hanson 2014), present a promising avenue for future research.

References

Ajello, A., L. Benzoni, and O. Chyruk. 2012. Core and "crust": Consumer prices and the term structure of interest rates. Working Paper, Federal Reserve Bank of Chicago.

Ang, A., and G. Bekaert. 2007. Stock return predictability: Is it there? *Review of Financial Studies* 20:651–707.

Ang, A., G. Bekaert, and M. Wei. 2007. Do macro variables, asset markets, or surveys forecast inflation better? *Journal of Monetary Economics* 54:1163–212.

———. 2008. The term structure of real rates and expected inflation. *Journal of Finance* 63:797–849.

Atkeson, A., and L. Ohanian. 2001. Are Phillips curves useful for forecasting inflation? *Federal Reserve Bank of Minneapolis Quarterly Review* 25:2–11.

Bansal, R., and I. Shaliastovich. 2013. A long-run risk explanation of predictability puzzles in bond and currency markets. *Review of Financial Studies* 26:1–33.

Barr, D., and J. Campbell. 1997. Inflation, real interest rate, and the bond market: A study of UK nominal and index-linked government bond prices. *Journal of Monetary Economics* 39:361–83.

Bekaert, G., S. Cho, and A. Moreno. 2010. New Keynesian macroeconomics and the term structure. *Journal of Money, Credit and Banking* 42:34–62.

Bekaert, G., R. Hodrick, and D. Marshall. 1997. On biases in tests of the expectations hypothesis of the term structure of interest rates. *Journal of Financial Economics* 44:309–48.

Blinder, A., and R. Reis. 2005. Understanding the Greenspan standard. In *The Greenspan era: Lessons for the future*. Kansas City, MO: Federal Reserve Bank of Kansas City.

Branch, W., and G. Evans. 2006. A simple recursive forecasting model. *Economic Letters* 91:158–66.

Brayton, F., and P. Tinsley. 1996. *A guide to FRB/US: A macroeconomic model of the United States*. Washington, DC: Macroeconomic and Quantitative Studies, Federal Reserve Board.

Campbell, J. 2006. Comment on equilibrium yield curves. *NBER Macro Annual* 21:459–68.

Campbell, J., and R. Shiller. 1991. Yield spreads and interest rate movements: A bird's eye view. *Review of Economic Studies* 58:495–514.

Campbell, J., A. Sunderam, and L. Viceira. 2011. Inflation bets or deflation hedges? The changing risk of nominal bonds. Working Paper, Harvard Business School.

Campbell, J., and S. Thompson. 2008. Predicting excess stock returns out of sample: Can anything beat the historical average? *Review of Financial Studies* 21:1509–31.

Clarida, R., J. Galí, and M. Gertler. 2000. Monetary policy rules and macroeconomic stability: Evidence and some theory. *Quarterly Journal of Economics* 115:147–80.

Clark, T., and M. McCracken. 2001. Tests of equal forecast accuracy and encompassing for nested models. *Journal of Econometrics* 105:85–110.

Clark, T., and T. Nakata. 2008. Has the behavior of inflation and long-term inflation expectations changed? *Federal Reserve Bank of Kansas City, Economic Review* QI:17–50.

Cochrane, J., and M. Piazzesi. 2005. Bond risk premia. *American Economic Review* 95:138–60.

———. 2008. Decomposing the yield curve. Working Paper, University of Chicago.

Cogley, T. 2002. A simple adaptive measure of core inflation. *Journal of Money, Credit and Banking* 34:94–113.

Coibion, O., and Y. Gorodnichenko. 2011. Monetary policy, trend inflation and the great moderation: An alternative interpretation. *American Economic Review* 101:341–70.

Croushore, D., and T. Stark. 1999. A real-time data set for macroeconomists. Working Paper, FRB Philadelphia.

Crowder, W., and D. Hoffman. 1996. The long-run relationship between nominal interest rates and inflation: The Fisher equation revisited. *Journal of Money, Credit and Banking* 28:102–18.

Duffee, G. 2010. Sharpe ratios in term structure models. Working Paper, Johns Hopkins University.

———. 2011. Information in (and not in) the term structure. *Review of Financial Studies* 24:2895–934.

———. 2012. Forecasting interest rates In *Handbook of economic forecasting*, 385–426. Elsevier.

———. 2013. Bond pricing and the macroeconomy In *Handbook of the economics of finance*, 907–67. Elsevier.

Evans, G., S. Honkapohja, and N. Williams. 2010. Generalized stochastic gradient learning. *International Economic Review* 51:237–62.

Evans, M., and P. Wachtel. 1992. Interpreting the movements in short-term interest rates. *Journal of Business* 65:395–429.

Fama, E. 1990. Term-structure forecasts of interest rates, inflation, and real returns. *Journal of Monetary Economics* 25:59–76.

———. 2006. The behavior of interest rates. *Review of Financial Studies* 19:359–79.

Fama, E., and R. Bliss. 1987. The information in long-maturity forward rates. *American Economic Review* 77:680–92.

Faust, J., and J. Wright. 2013. Forecasting inflation In *Hanbook of economic forecasting*, 3–51. Elsevier.

Greenwood, R., and A. Shleifer. 2013. Expectations of returns and expected returns. *Review of Financial Studies* 27:714–46.

Gürkaynak, R., B. Sack, and J. Wright. 2006. The U.S. Treasury yield curve: 1961 to the present. Working Paper, Federal Reserve Board.

Hanson, S. 2014. Mortgage convexity. *Journal of Financial Economics* 113:270–99.

Hodrick, R. 1992. Dividend yields and expected stock returns: Alternative procedures for inference and measurement. *Review of Financial Studies* 5:357–86.

Inoue, A., and L. Kilian. 2004. In-sample or out-of-sample tests of predictability: Which one should we use? *Econometric Reviews* 23:371–402.

Ireland, P. 1996. Long-term interest rates and inflation: A Fisherian approach. *Federal Reserve Bank of Richmond Economic Quarterly* 82:21–36.

Kozicki, S., and P. Tinsley. 2001. Shifting endpoints in term structure of interest rates. *Journal of Monetary Economics* 47:613–52.

———. 2005. Permanent and transitory policy shocks in an empirical macro model with asymmetric information. *Journal of Economic Dynamics and Control* 29:1985–2015.

Krishnamurthy, A., and A. Vissing-Jorgensen. 2012. The aggregate demand for Treasury debt. *Journal of Political Economy* 120:233–67.

Le, A., and K. Singleton. 2013. The structure of risks in equilibrium affine models of bond yields. Working Paper, University of North Carolina at Chapel Hill and Stanford University.

Lettau, M., and J. Wachter. 2011. The term structures of equity and interest rates. *Journal of Financial Economics* 101:90–113.

Li, H., and G. Maddala. 1997. Bootstrapping cointegrating regressions. *Journal of Econometrics* 80:297–318.

Mishkin, F. 2007. Headline versus core inflation in the conduct of monetary policy. Speech at the Business Cycles, International Transmission and Macroeconomic Policies Conference, HEC Montreal, Montreal, Canada.

Orphanides, A. 2004. Monetary policy rules, macroeconomic stability and inflation: A view from the trenches. *Journal of Money, Credit and Banking* 36:151–75.

Orphanides, A., and J. Williams. 2005. The decline of activist stabilization policy: Natural rate misperceptions, learning, and expectations. *Journal of Economic Dynamics and Control* 29:1927–50.

Piazzesi, M., and M. Schneider. 2006. Equilibrium yield curves. *NBER Macro Annual* 21:389–442.

Rudebusch, G., and T. Wu. 2008. A macro-finance model of the term structure, monetary policy, and the economy. *The Economic Journal* 118:906–26.

Sargent, T. 1999. *The conquest of American inflation*. Princeton University Press.

Shome, D., S. Smith, and J. Pinkerton. 1988. The purchasing power of money and nominal interest rates: A re-examination. *Journal of Finance* 43:1113–25.

Stock, J., and M. Watson. 2011. Modeling inflation after the crisis. FRB Kansas City Symposium, Jackson Hole, Wyoming, August 26–28.

Thornton, D., and G. Valente. 2012. Out-of-sample predictions of bond excess returns and forward rates: An asset allocation perspective. *Review of Financial Studies* 25:3141–68.

Wei, M., and J. Wright. 2013. Reverse regressions and long-horizon forecasting. *Journal of Applied Econometrics* 28:353–71.

Wooldridge, J. 2010. *Econometric analysis of cross section and panel data*. The MIT Press.